CS 498: Machine Learning System Spring 2026

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Recall: Adaptive KV Cache

- Heavy Hitter (H2O): not all KV cache (tokens) are equally important
  - Keep the ones with high attention score to cap memory usage

- Attention Sink:
  - Keep the first several tokens and recent tokens
  - Highly effective yet hard to interpret

Agenda: Quantization

Inference Optimization (Part 8)

- LLM Quantization

Learning Objective:

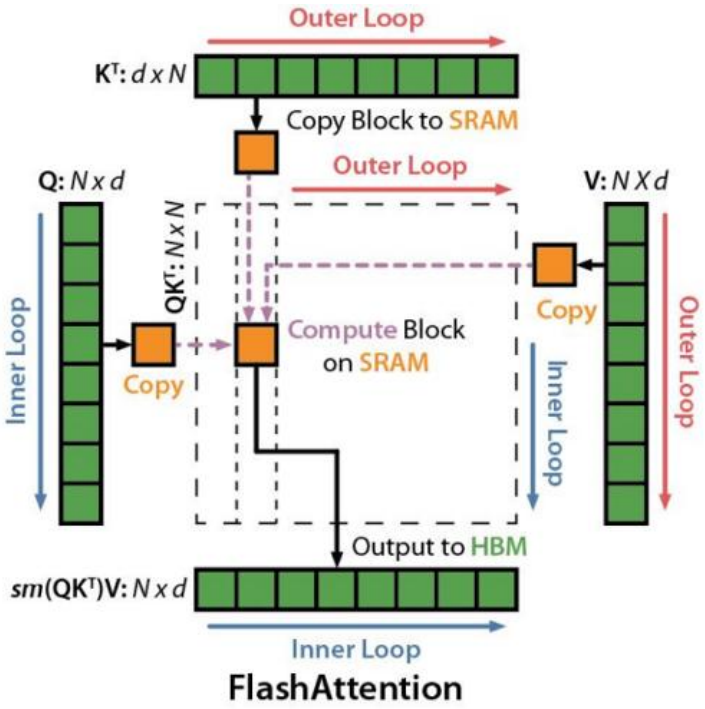
- Understand how LLM quantization reduces memory and compute cost
- Learn the core ideas behind SmoothQuant
- Analyze the trade-offs in accuracy and efficiency introduce by quantization

Reminder: Lab Assignment 3 due on April 26th

Gap Between the Supply and Demand of LLMs

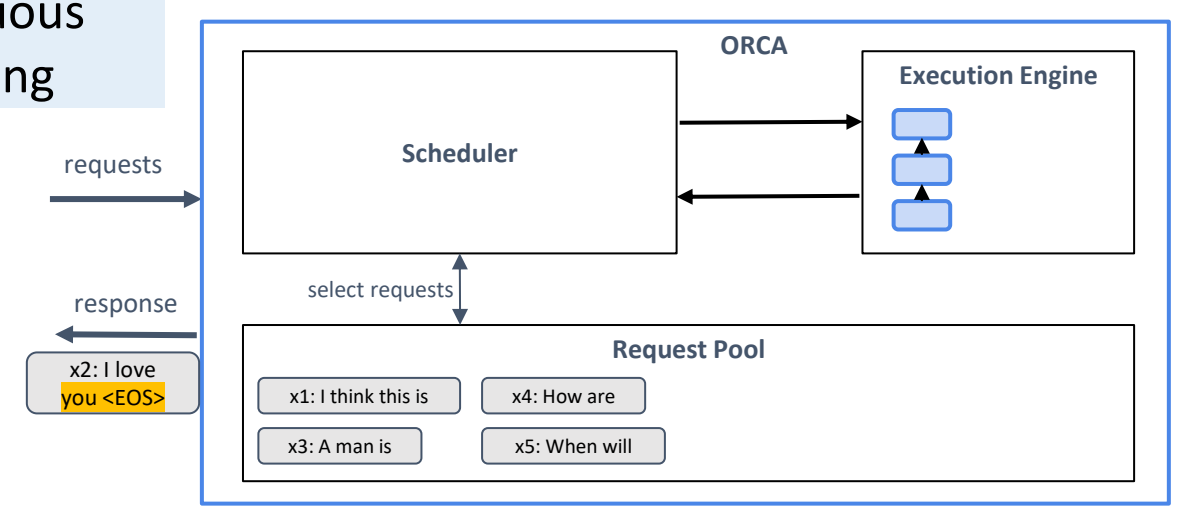
- Size of LLMs is increasing faster than GPU memory: **exponential growth in model size** vs. linear increase in GPU memory
- **Memory challenges:** 175B parameters in GPT-3 requires 350GB of memory to store just weights
- **Deployment difficulties:** Requires multiple GPUs, high latency, and resource intensive setups

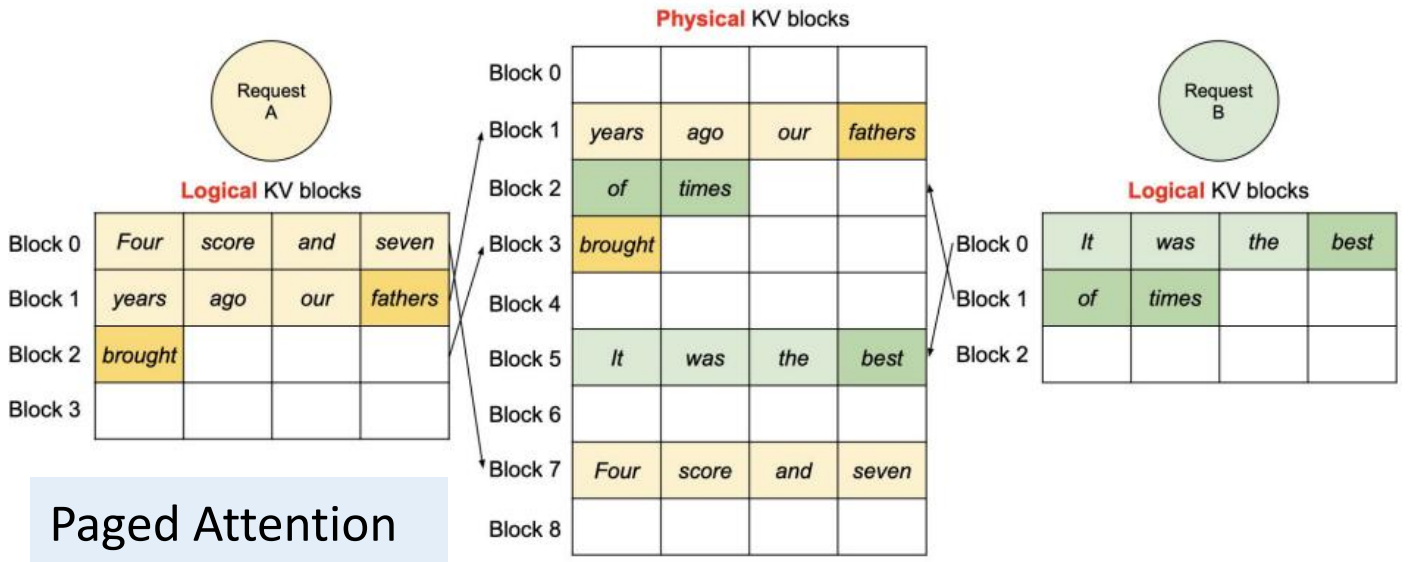
Do Previously Introduced Techniques Reduce LLM Memory?



Flash Attention

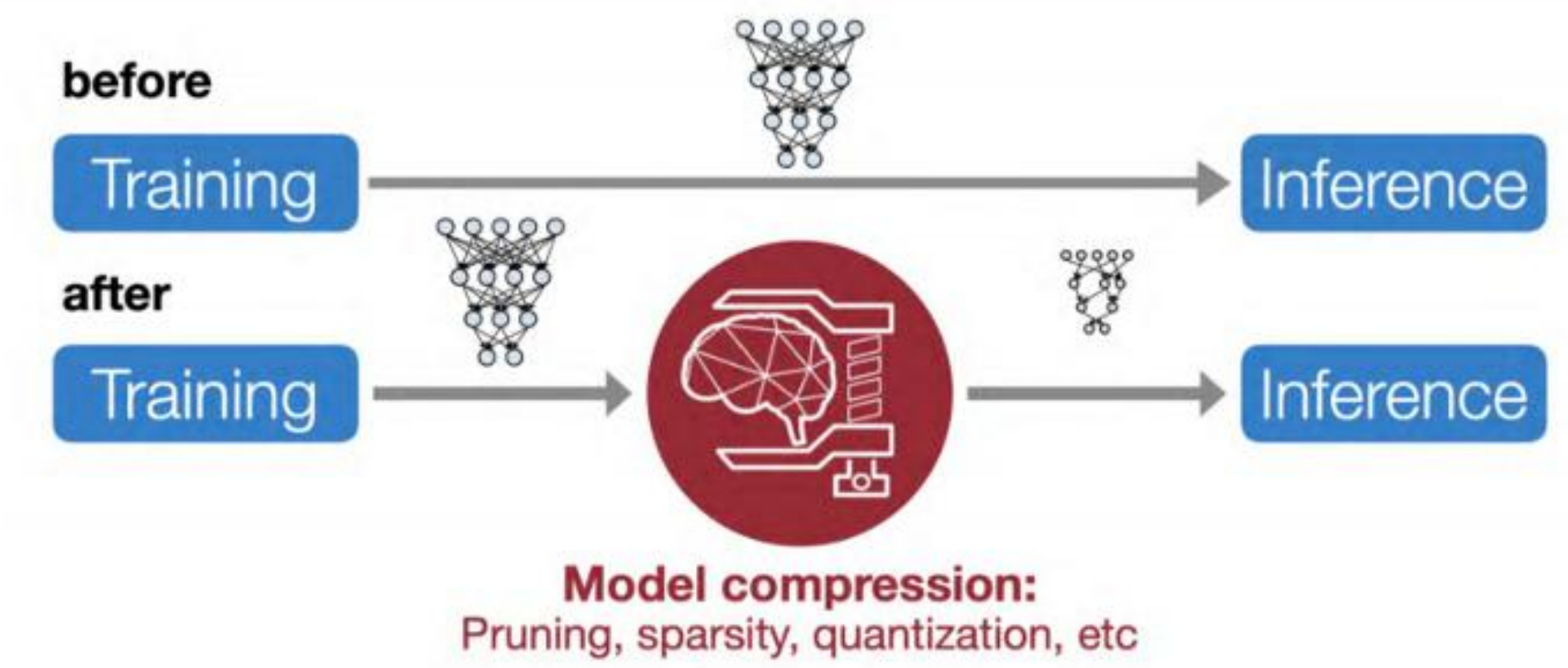
Continuous Batching





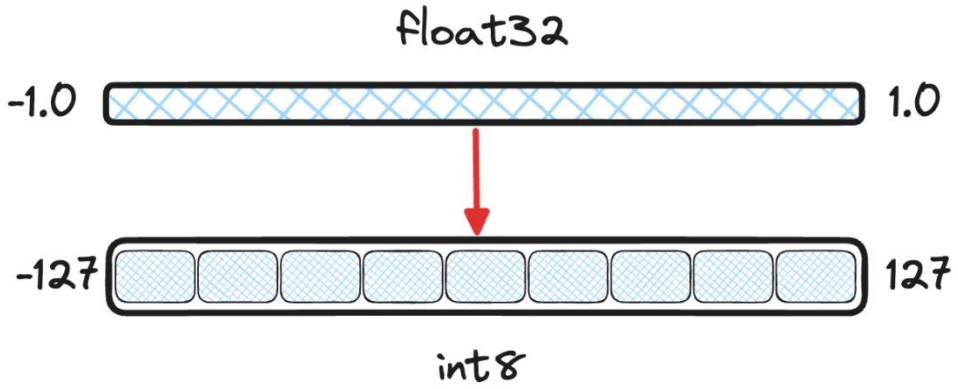
Paged Attention

Bridge the Gap through Model Compression



Quantization

- *Quantization is the process of mapping a large set of continuous or high-precision values (typically floating-point numbers) to a smaller set of discrete values (typically lower-precision integers) in order to reduce the computational and memory requirements of a model.*



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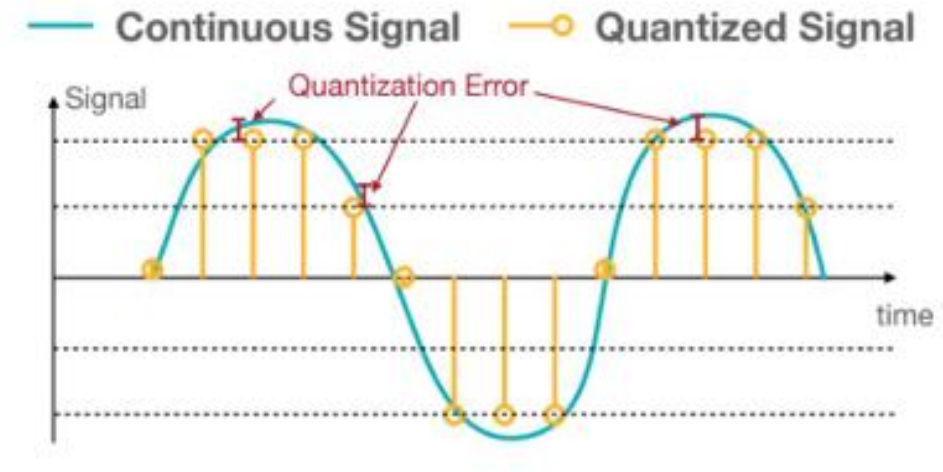

The difference between an input value and its quantized value is referred to as quantization error.

Originate from signal processing and information theory

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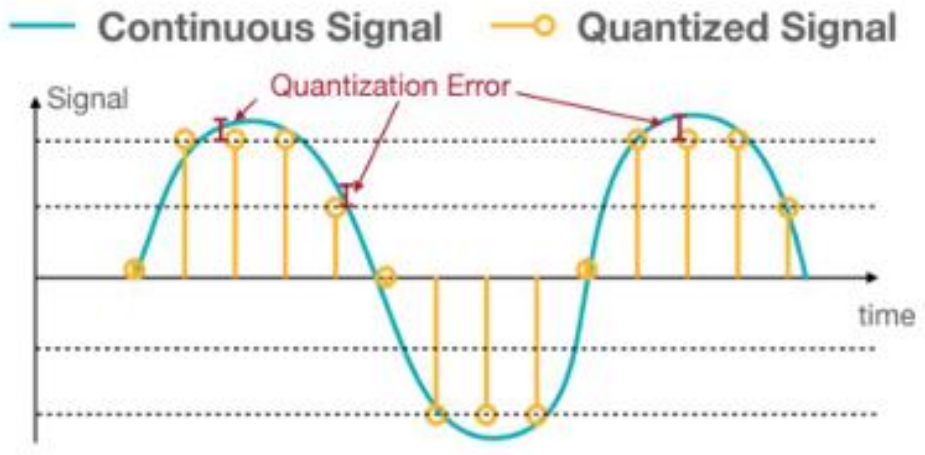

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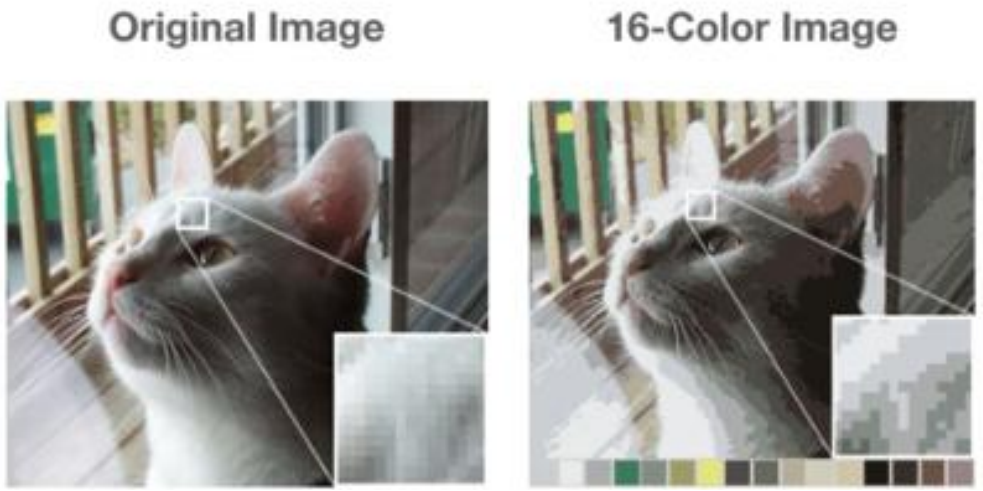

Originate from signal processing and information theory

Adapted to computer vision models in ML

Motivation for LLM Quantization

- LLMs often come in high precision formats such as FP16/BF16/FP32 and 10B → 1T+ parameters
  - Significant GPU memory requirements (memory capacity and bandwidth)

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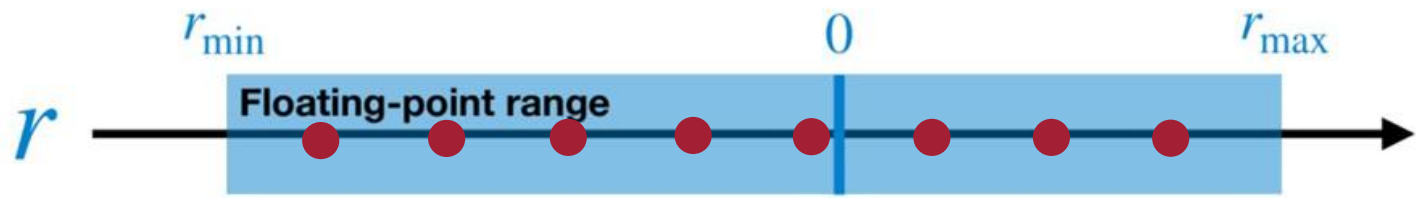
Mixed-Precision Training (Training Series, Part 6)

Quantization: Problem

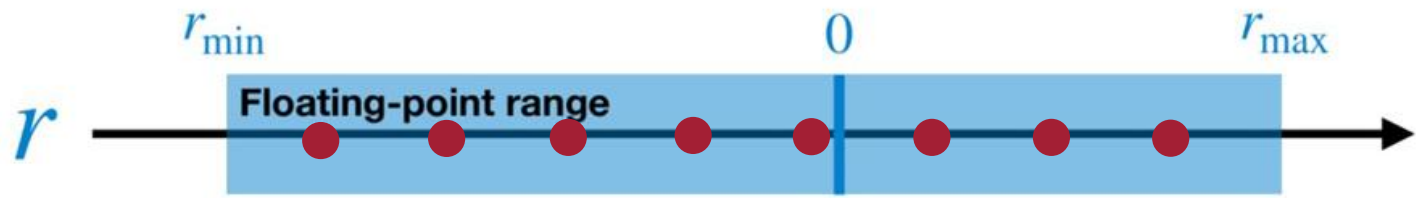
Quantization: Constraints



Question: Can we just round to integers?

Bit Width	q_{min}	q_{max}
2	-2	1
3	-4	3
4	-8	7
N	-2^{N-1}	$2^{N-1}-1$

Quantization: Constraints



Question: Can we just round to integers?

Massive information loss

Bit Width	q_{min}	q_{max}
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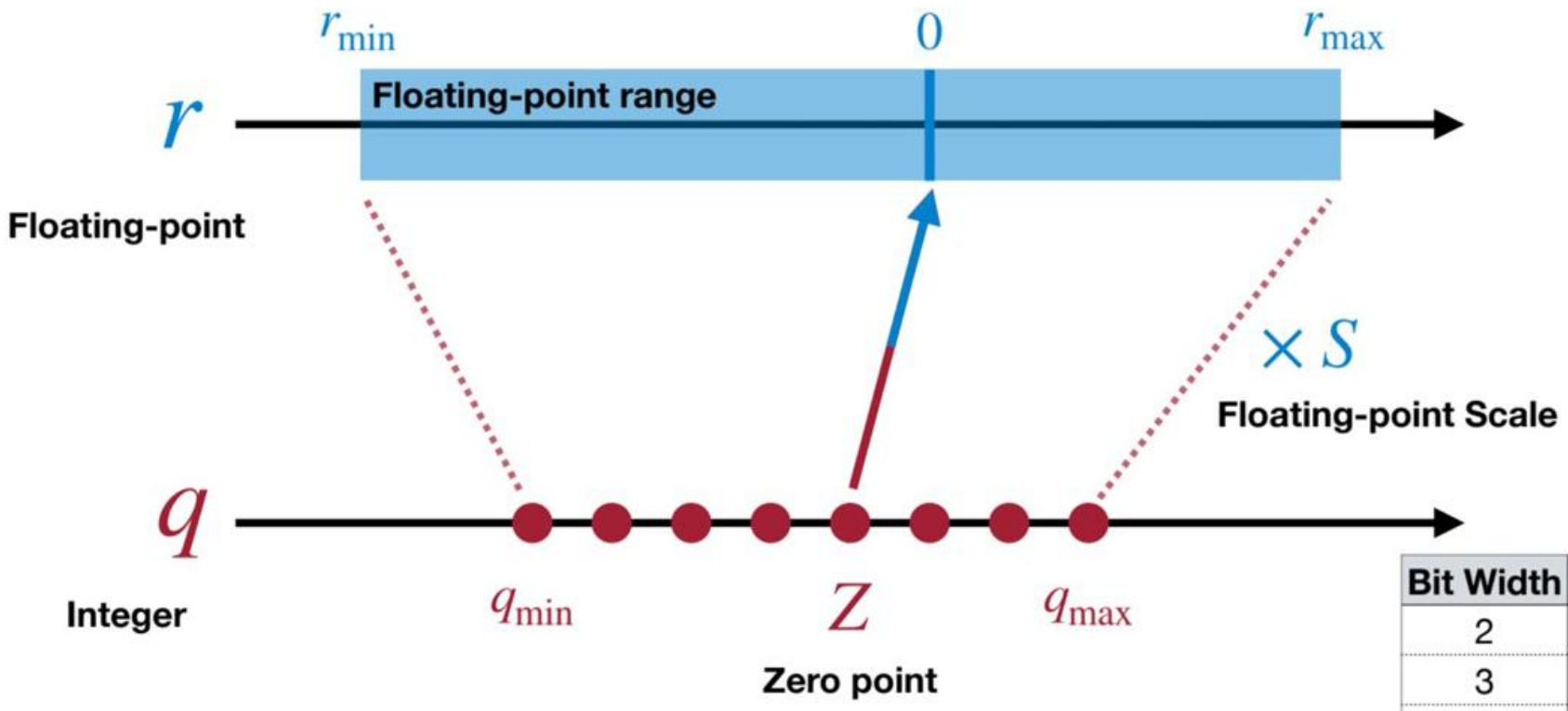
N	-2^{N-1}	$2^{N-1}-1$

Quantization: Linear Quantization + Round-to-Nearest

- An affine mapping of integers to real numbers $r = S(q - Z)$



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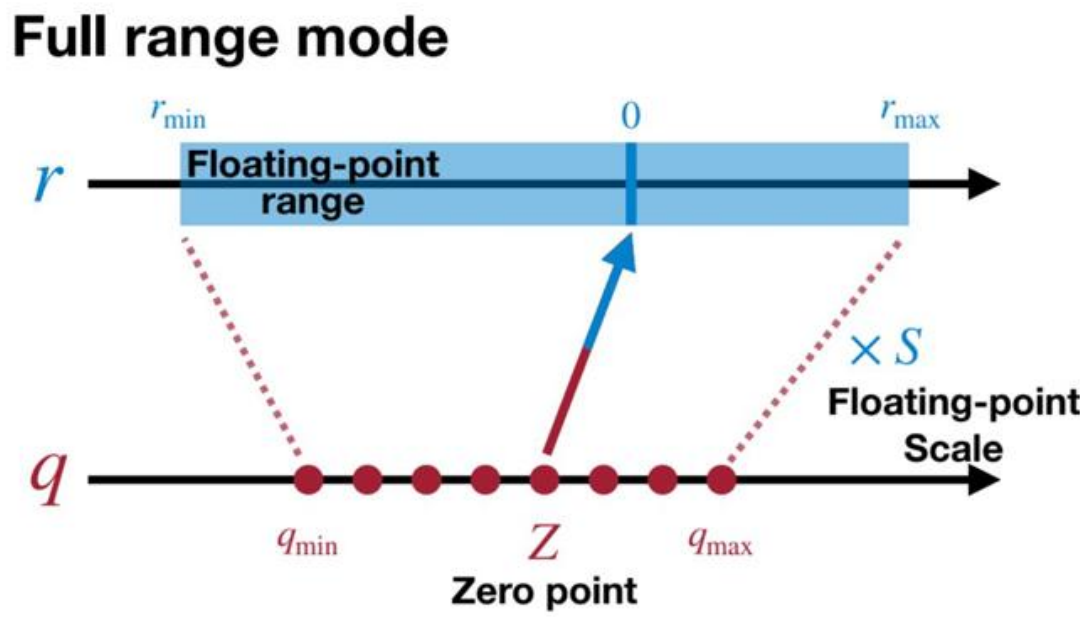

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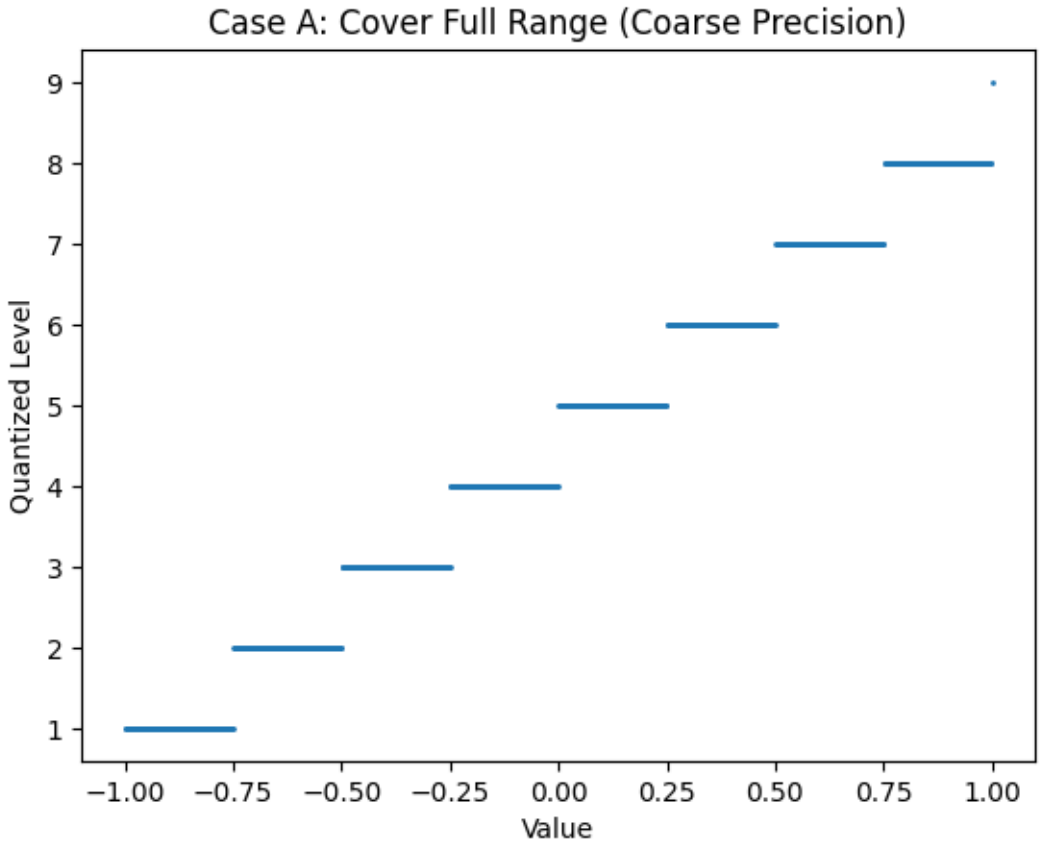
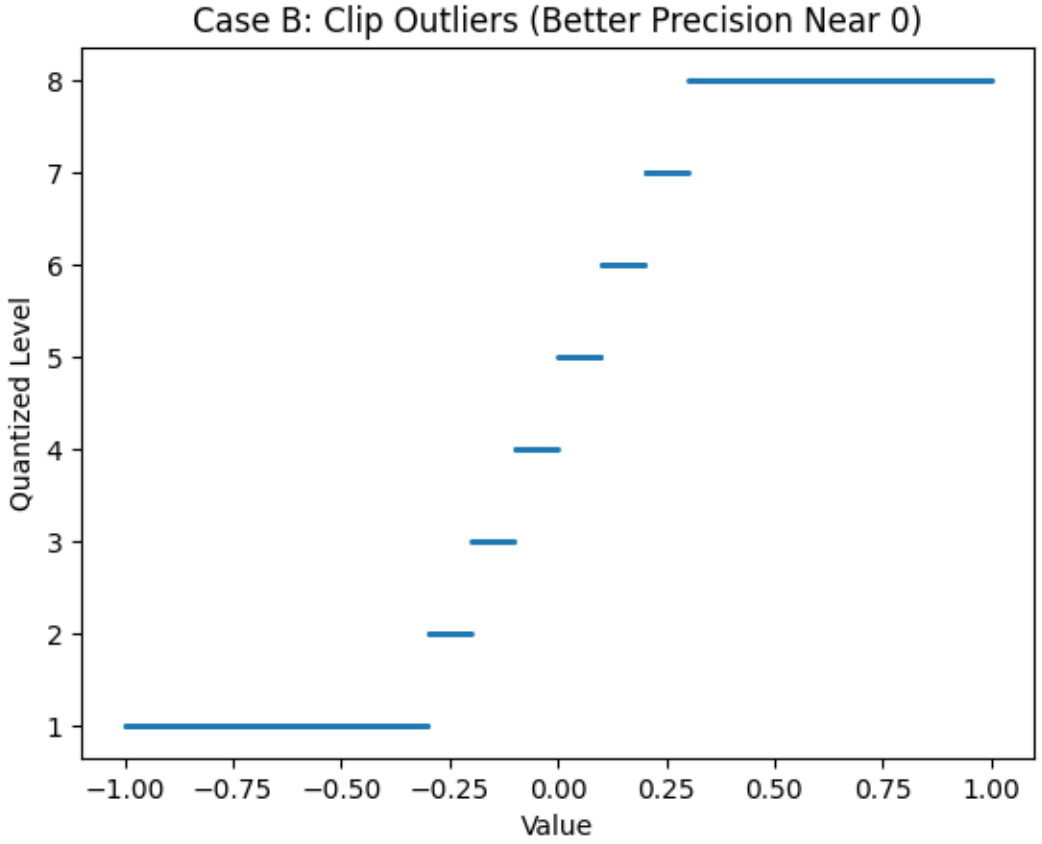
Without scaling, most values would collapse to 0, so scaling is necessary, but how?

Quantization: Range vs. Precision Tradeoff



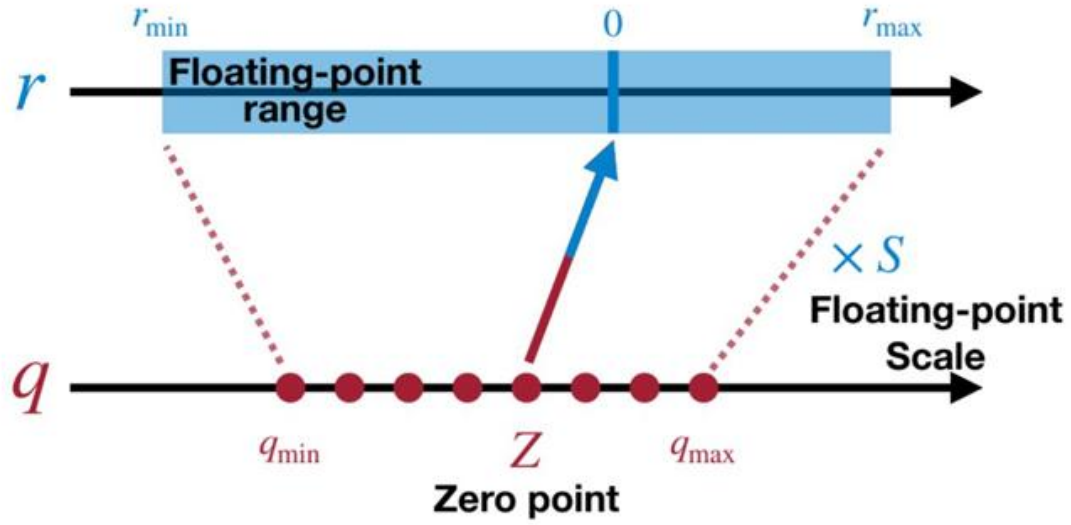
Full range coverage
Large bucket size
Low precision near 0

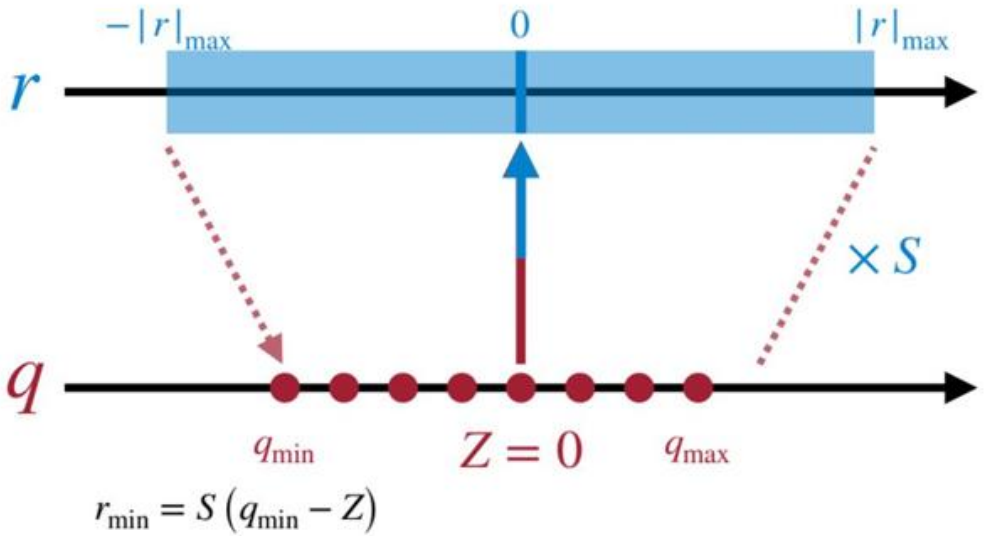


Partial range (outliers clipped)
Small bucket size
Better precision near 0

Quantization: Asymmetric vs. Symmetric

Full range mode





$$r_{min} = S\,(q_{min} - Z)$$

Bit Width	q_{min}	q_{max}
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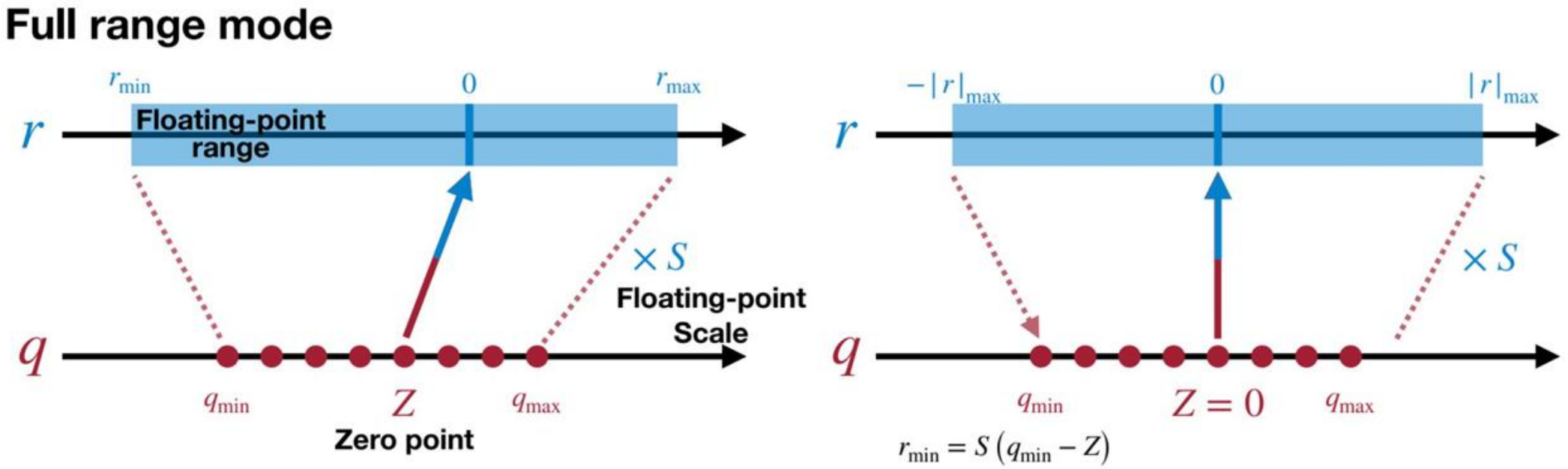
N	-2^{N-1}	$2^{N-1}-1$

$$S = \frac{r_{max} - r_{min}}{q_{max} - q_{min}}$$

$$S = \frac{r_{min}}{q_{min} - Z} = \frac{-|r|_{max}}{q_{min}} = \frac{|r|_{max}}{2^{N-1}}$$

Quantization: Asymmetric vs. Symmetric



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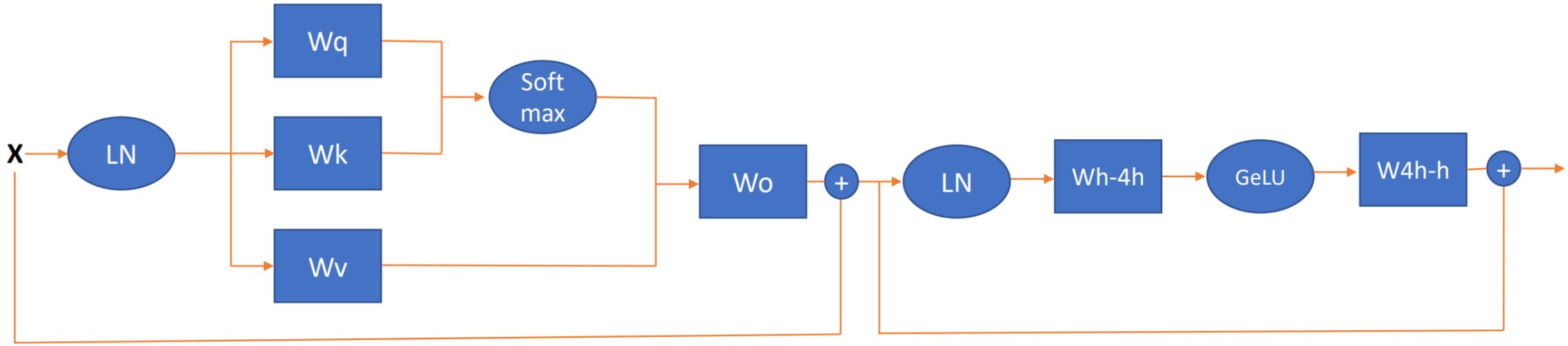
Next: How to apply quantization to LLMs

LLM Quantization

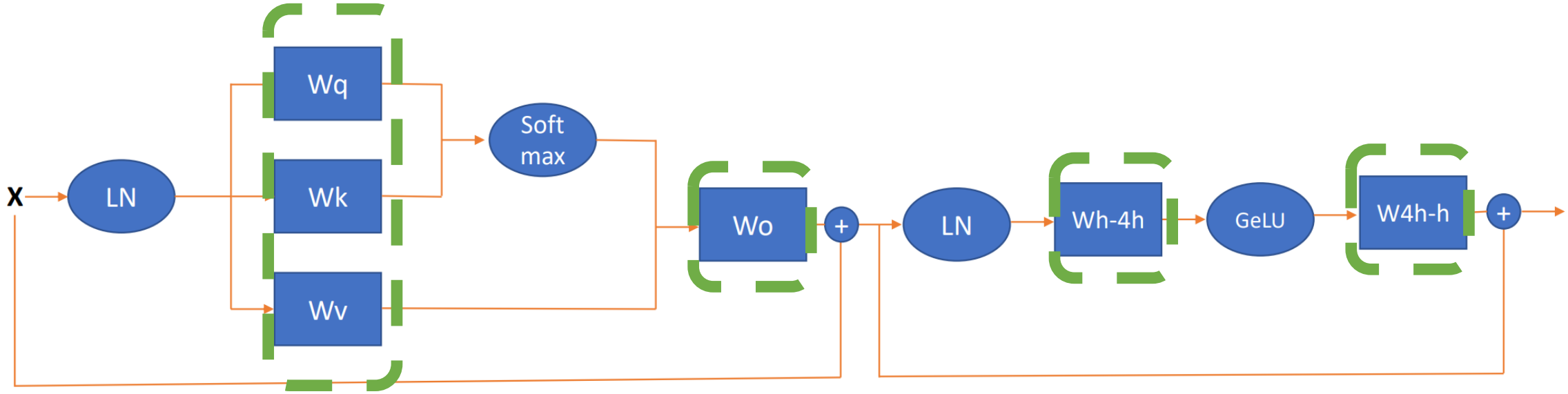


- 8-bit quantization

$$\mathbf{x}_{quantize} = round\left(clamp(\frac{\mathbf{x}}{S}, -2^{bit-1}, 2^{bit-1}-1)\right)$$

Where can we quantize without breaking the model?

LLM 8-bit Quantization: Weights



- 8-bit weight quantization

$$\mathbf{x}_{quantize} = round\left(clamp(\frac{\mathbf{x}}{S}, -2^{bit-1}, 2^{bit-1}-1)\right)$$

FP32 weight matrix

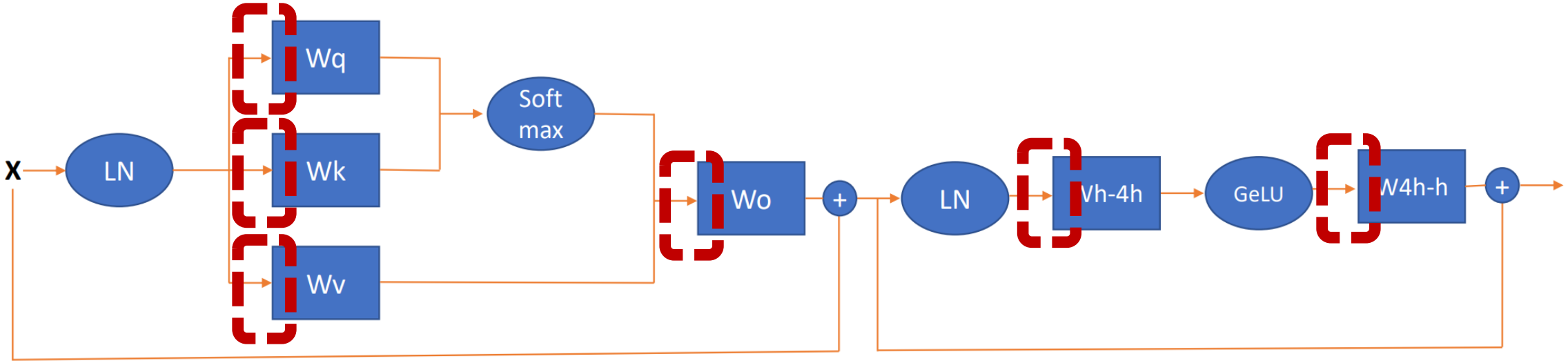
1.1	2.2	0.1	-0.1	-5.5	-6.6
…					
…					
…					
…					
1.1	2.1	0.1	-0.1	-4.8	-6.6

Scaling Factor $1/S$ ≈ 0.05 *

8-bit quantization

21	42	2	-2	-106	-127
…					
…					
…					
…					
21	40	2	-2	-92	-127

LLM 8-bit Quantization: Activation



- 8-bit activation
  (Input to the linear layer)

$$\mathbf{x}_{quantize} = round\left(clamp(\frac{\mathbf{x}}{S}, -2^{bit-1}, 2^{bit-1}-1)\right)$$

FP32 activation matrix

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Scaling Factor $1/S$ $\approx$ 0.05*

8-bit quantization

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…					
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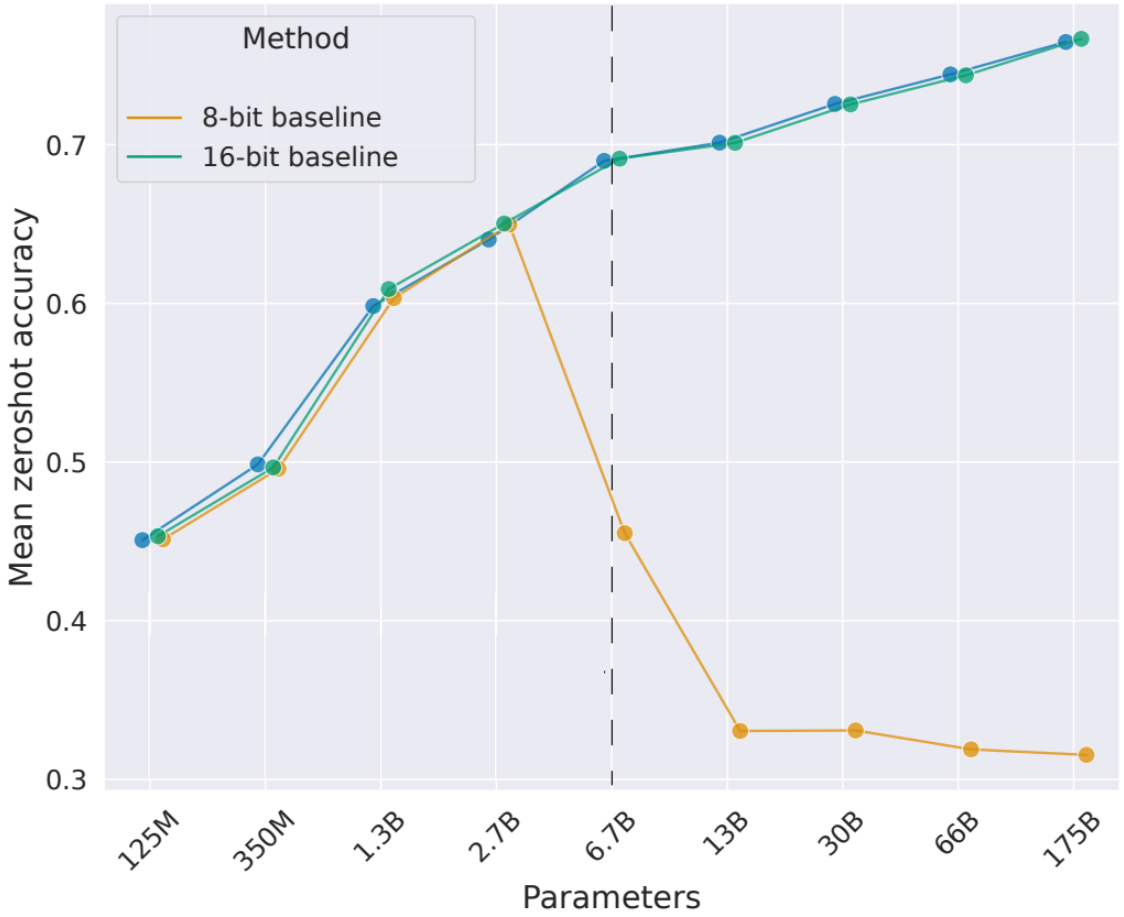
LLM Quantization Challenges

- Standard quantization strategy leads to catastrophic accuracy drop

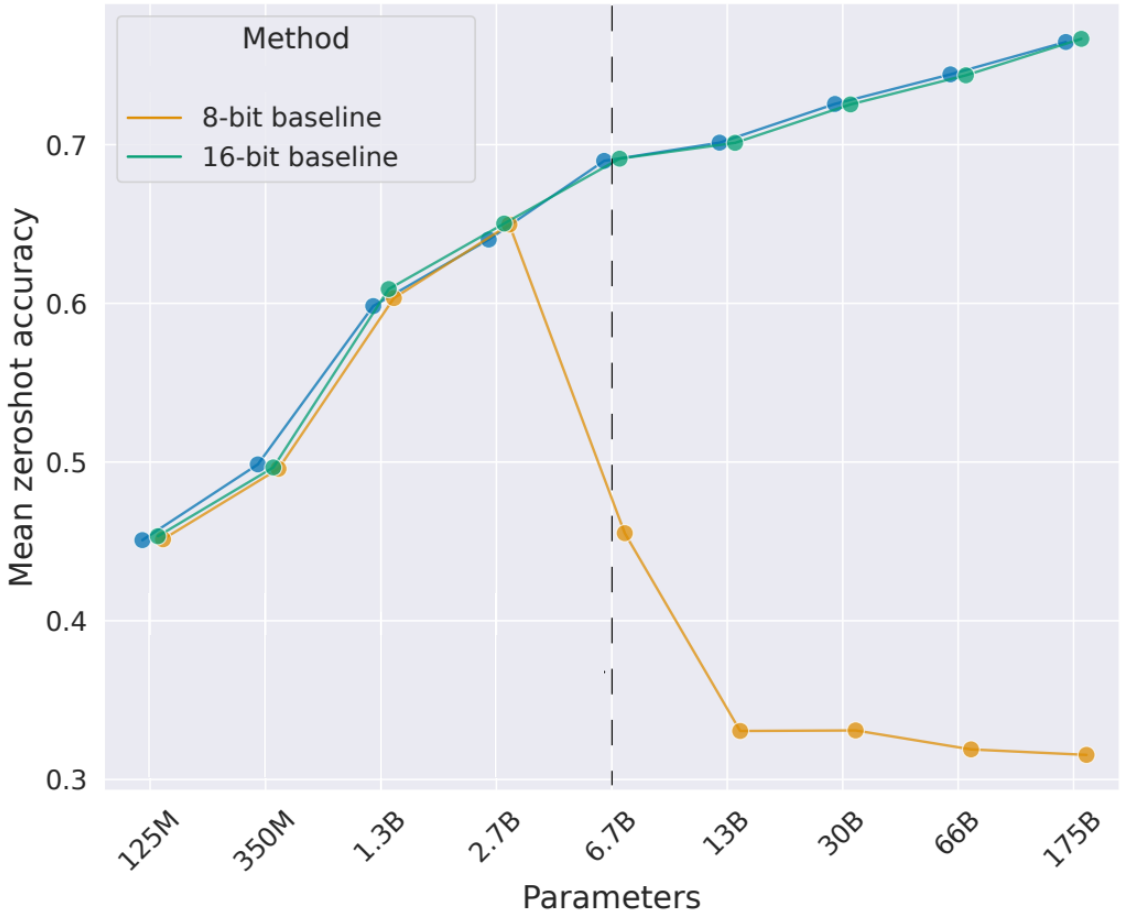


LLM.int8(): 8-bit Matrix Multiplication for Transformers at Scale, 2023

LLM Quantization Challenges

- Standard quantization strategy leads to catastrophic accuracy drop

Discussion: Something fundamentally changs in large LLMs. What changes as models get larger?



LLM.int8(): 8-bit Matrix Multiplication for Transformers at Scale, 2023

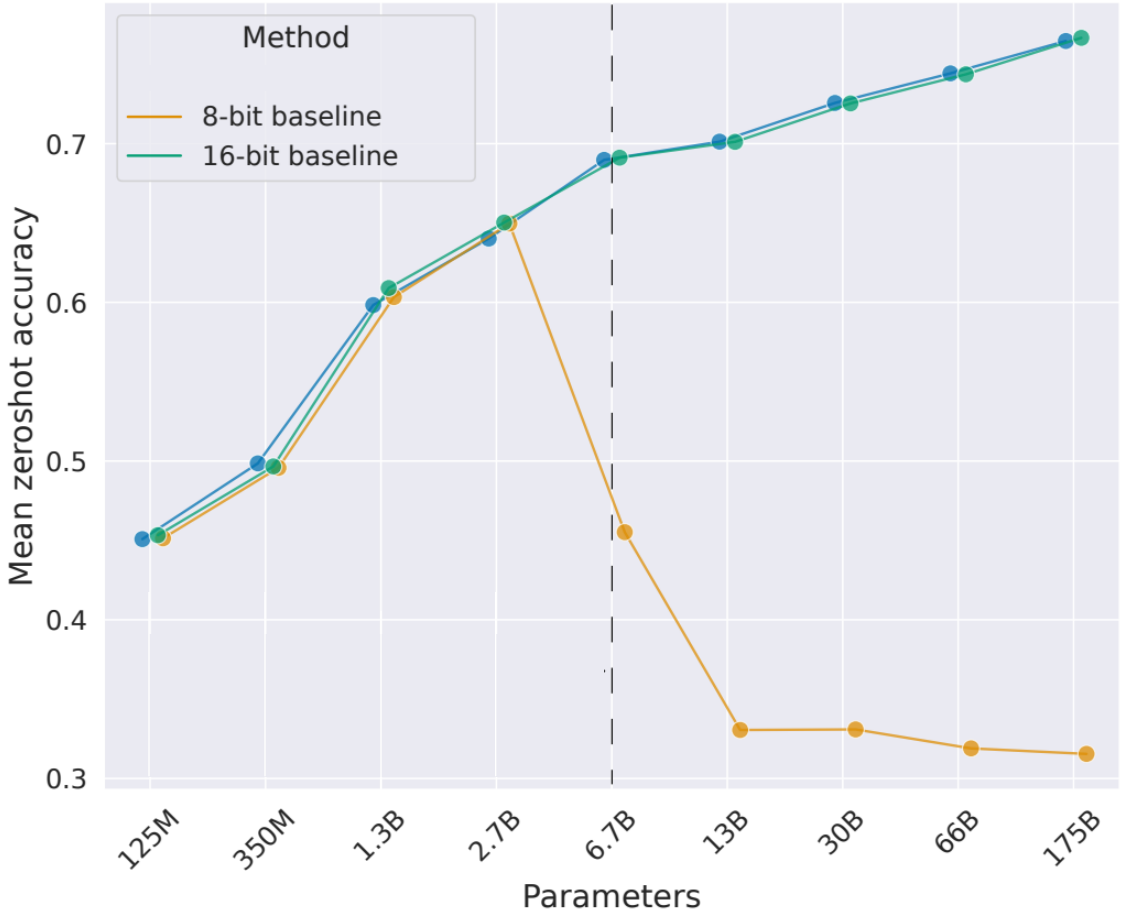
LLM Quantization Challenges

- Standard quantization strategy leads to catastrophic accuracy drop

Output = Activation X Weight

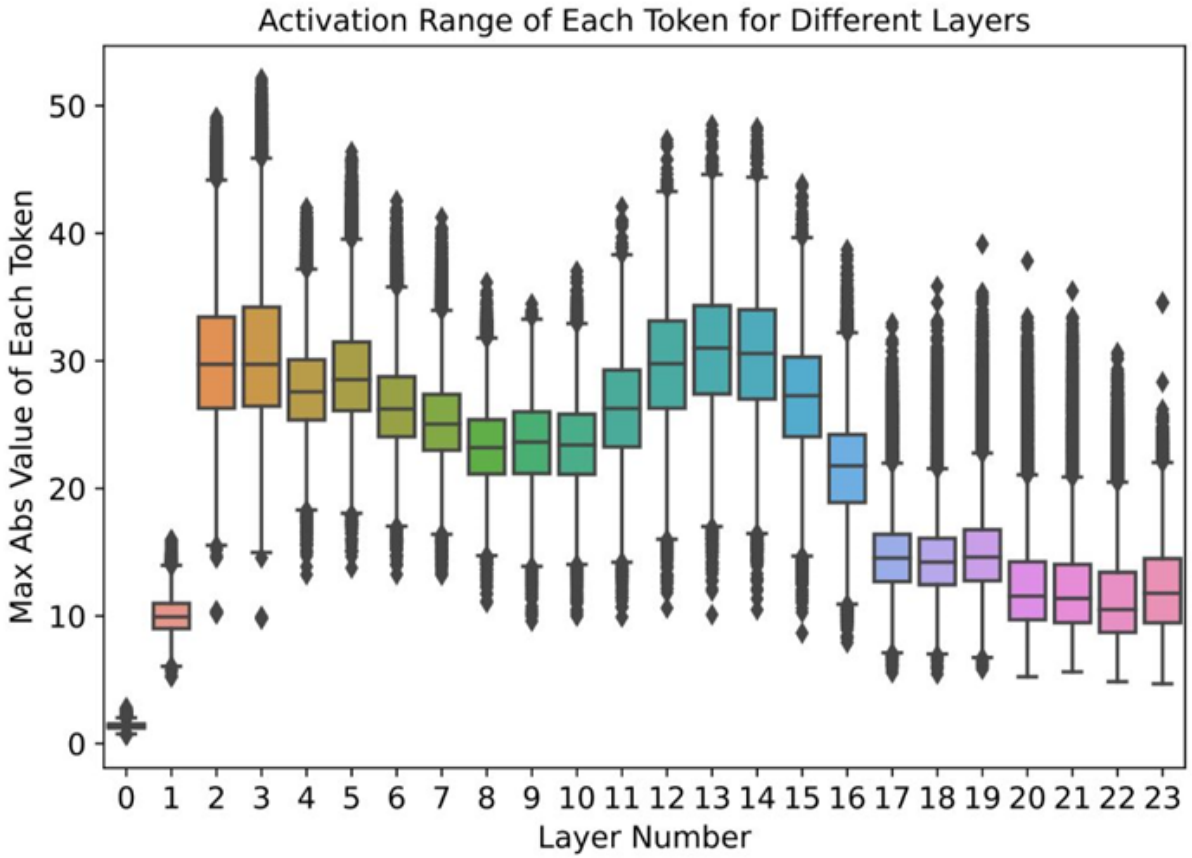
Precision	Lambada (↑)
W16A16	49.3
W8A16	49.3
W16A8	44.7
W8A8	42.6

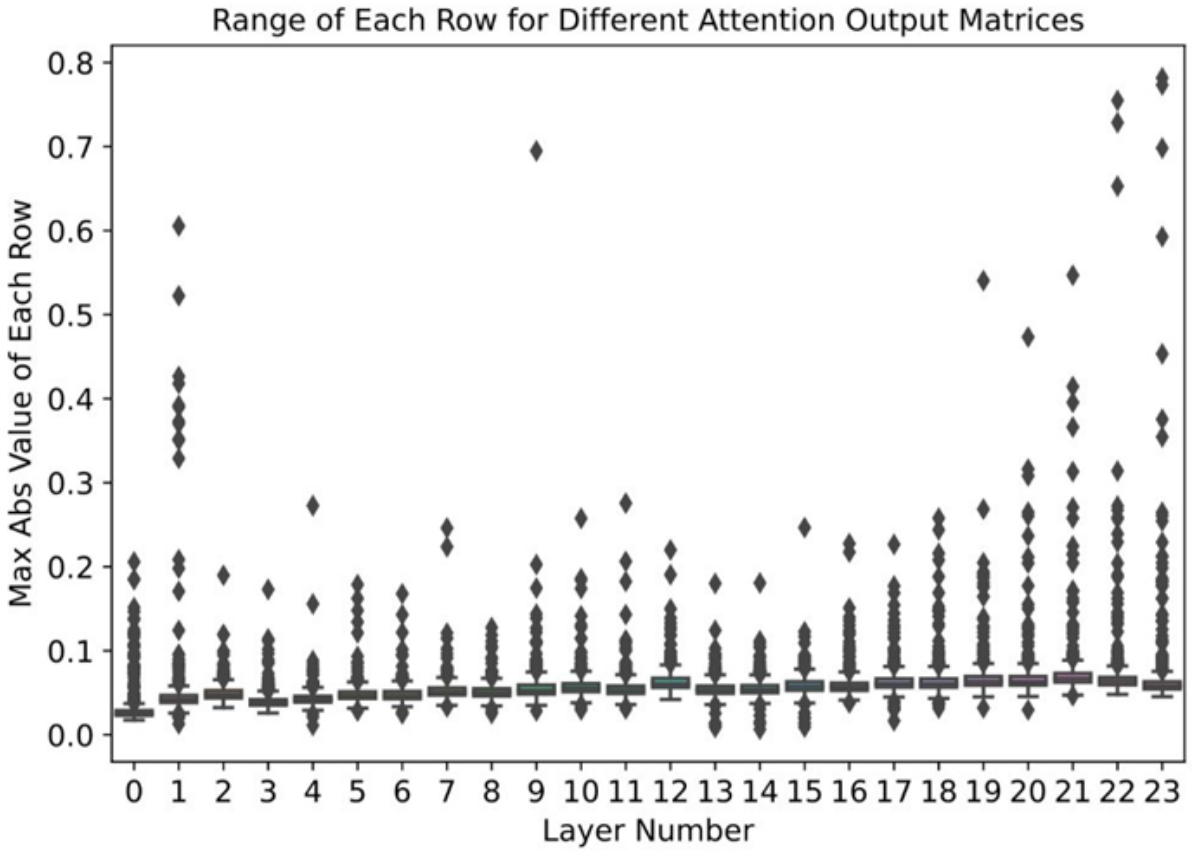
WxAy means x-/y-bit for weight/activation.



INT8 activation quantization causes the primary accuracy loss

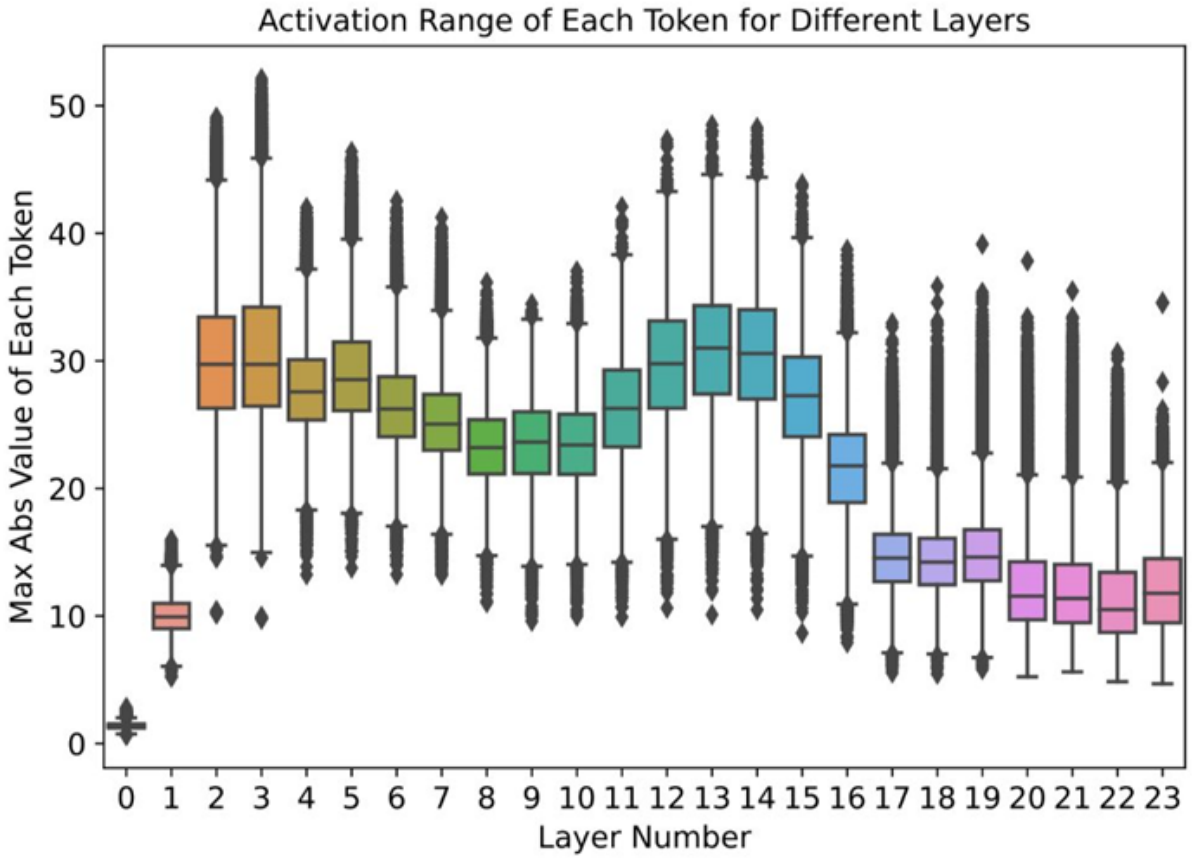
LLM Quantization Challenges

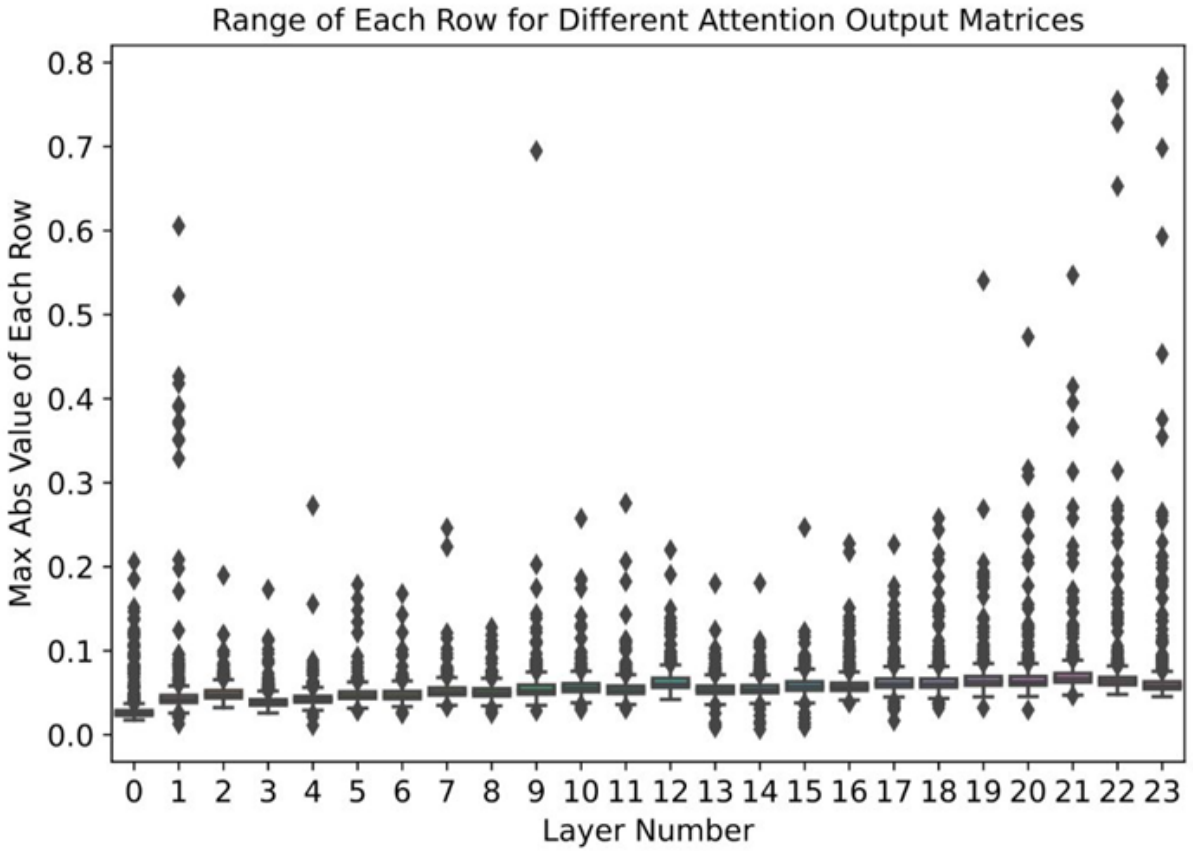




ZeroQuant: Efficient and Affordable Post-Training Quantization for Large-Scale Transformers, 2023

LLM Quantization Challenges



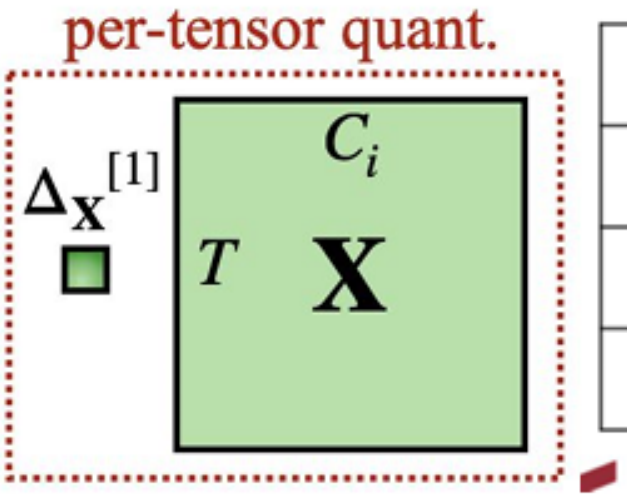


Questions: How do we mitigate accuracy loss from large dynamic range?

Quantization Can Have Different Granularities

- Per-Tensor Quantization
  - Uses a single scaling factor for the entire matrix

$$S = \frac{|r|_{\max}}{q_{\max}} = \frac{2.12}{2^{2-1} - 1} = 2.12$$



1	0	1	0
0	0	-1	1
0	1	0	0
1	0	1	1

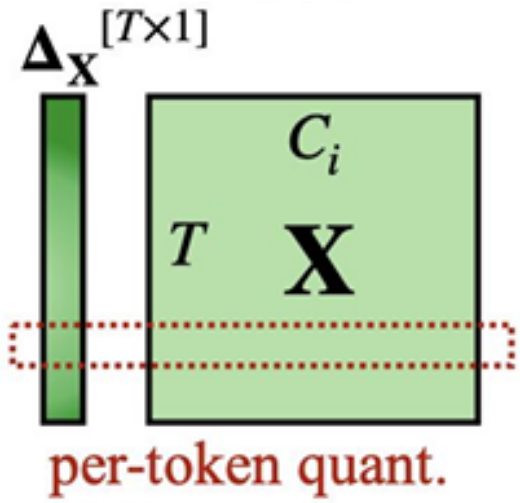
Quantized

2.12	0	2.12	0
0	0	-2.12	2.12
0	2.12	0	0
2.12	0	2.12	2.12

Reconstructed

Quantization Granularity

- Per-Tensor Quantization
- Per-Token Quantization
  - Uses different scaling factor for activations associated with each token
- Per-Channel (e.g., weight column) Quantization
- Group-wise Quantization



1	0	1	0
0	0	-1	1
0	1	0	-1
1	0	1	1

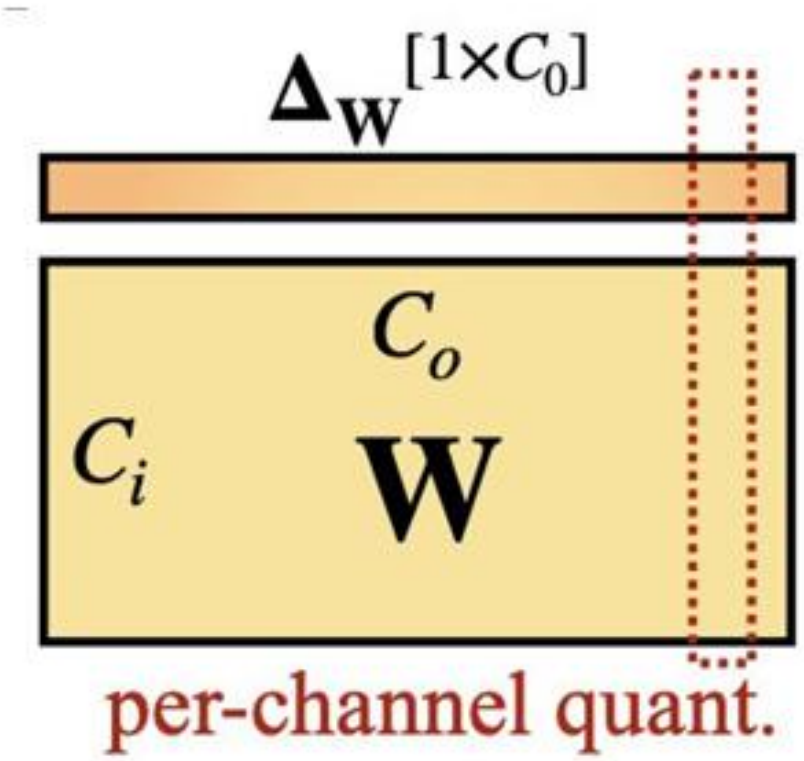
Quantized

2.09	0	2.09	0
0	0	-2.12	2.12
0	1.92	0	-1.92
1.87	0	1.87	1.87

Reconstructed

Quantization Granularity

- Per-Tensor Quantization
- Per-Token Quantization
- Per-Channel (e.g., weight column) Quantization
  - Uses different scaling factors for each weight channels
- Group-wise Quantization



Quantization Granularity

- Per-Tensor Quantization
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- Per-Channel (e.g., weight column) Quantization
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Model size (OPT-)	6.7B	13B	30B	66B	175B
FP16	64.9%	65.6%	67.9%	69.5%	71.6%
INT8 per-tensor	39.9%	33.0%	32.8%	33.1%	32.3%
INT8 per-token	42.5%	33.0%	33.1%	32.9%	31.7%
INT8 per-channel	64.8%	65.6%	68.0%	69.4%	71.4%

Per-channel QAT achieves best acc (tokens often demonstrate similar range patterns in the same channel) but hard to support in hardware

Quantization Granularity

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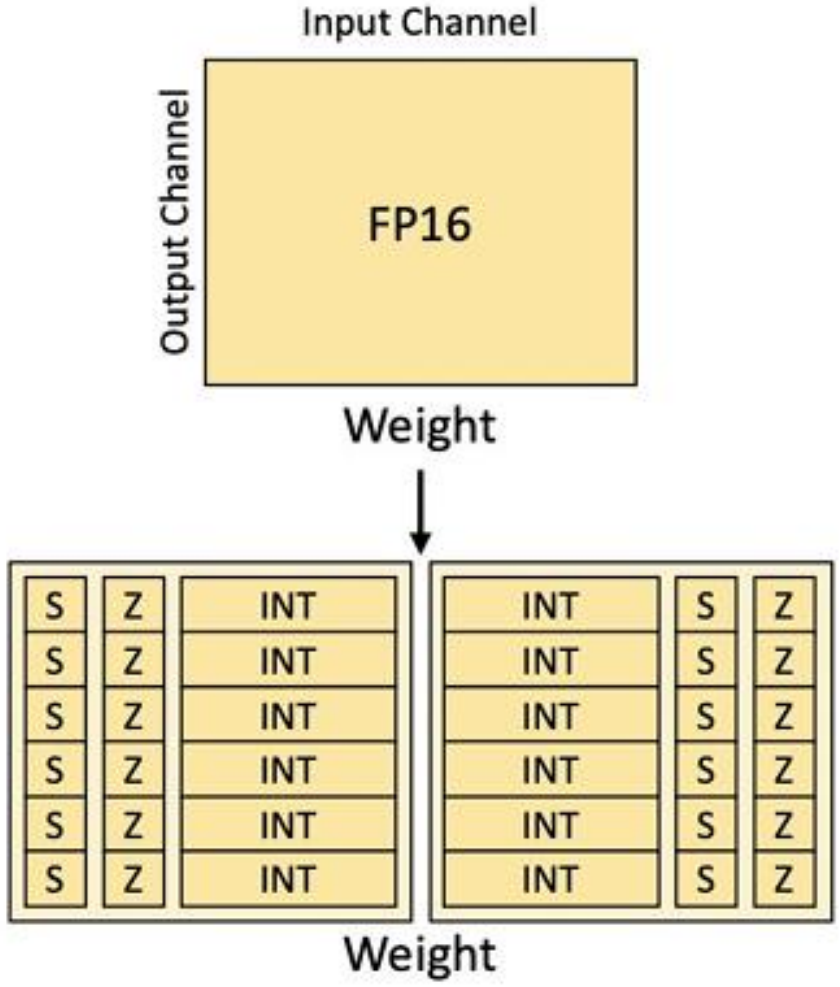
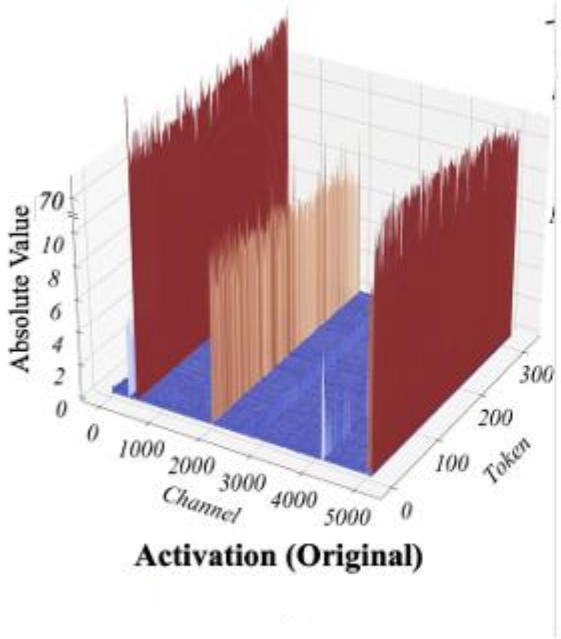
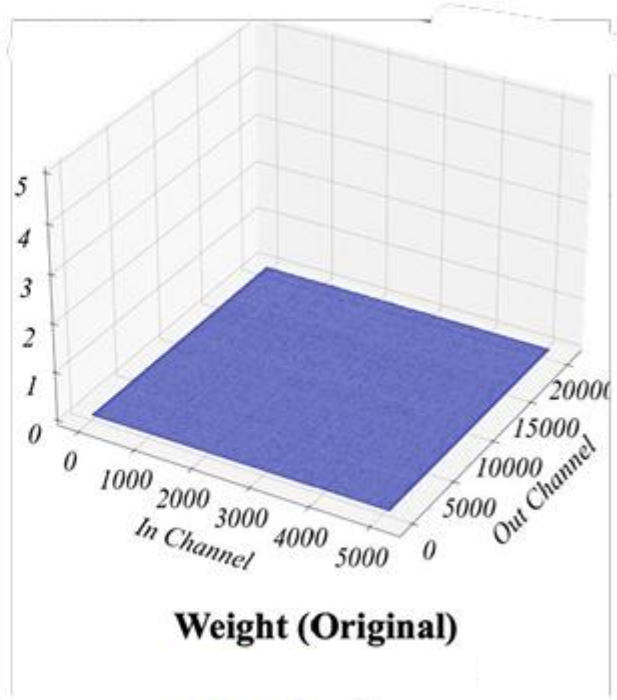
- Per-Token Quantization
- Per-Channel (e.g., weight column) Quantization
- Group-wise Quantization
  - Different scaling factors for different channels groups



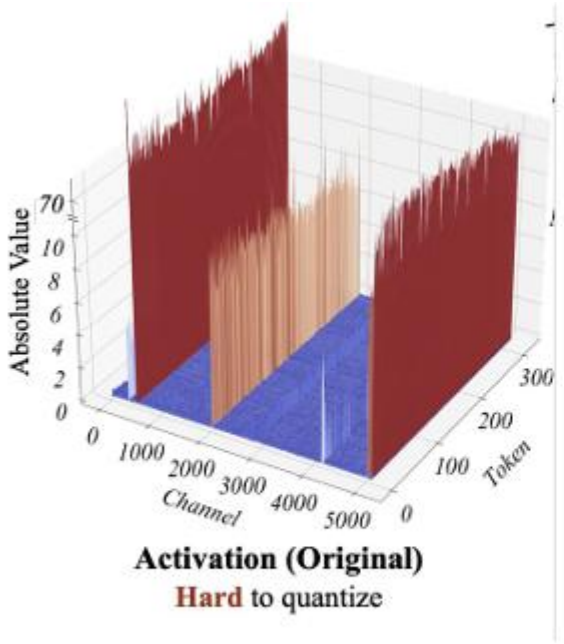
More Observations

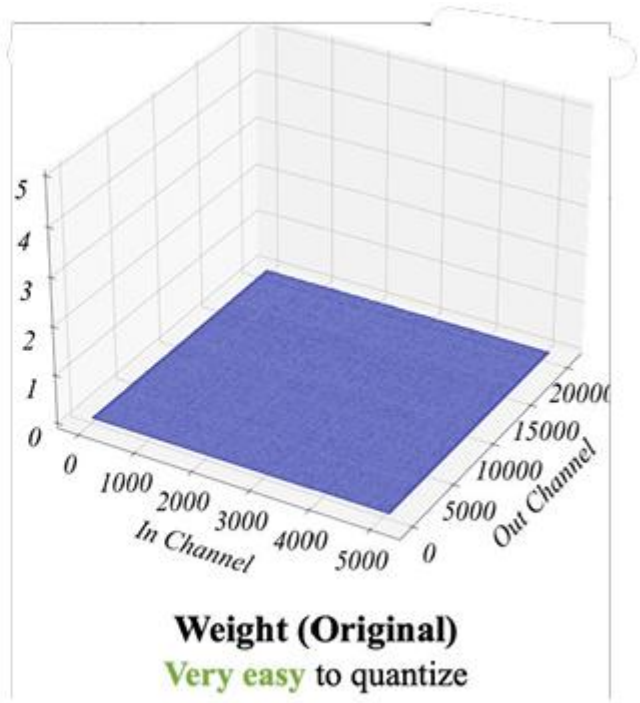




- Output = Activation X Weight, so we want to quantize both weights and activations into INT8 (i.e., W8A8) to utilize the integer kernels (e.g., INT8 GEMM)
- Activation has many outliers (10-70x larger) while the weight value is smooth (flat)
  - Outliers leave few effective bits for most values

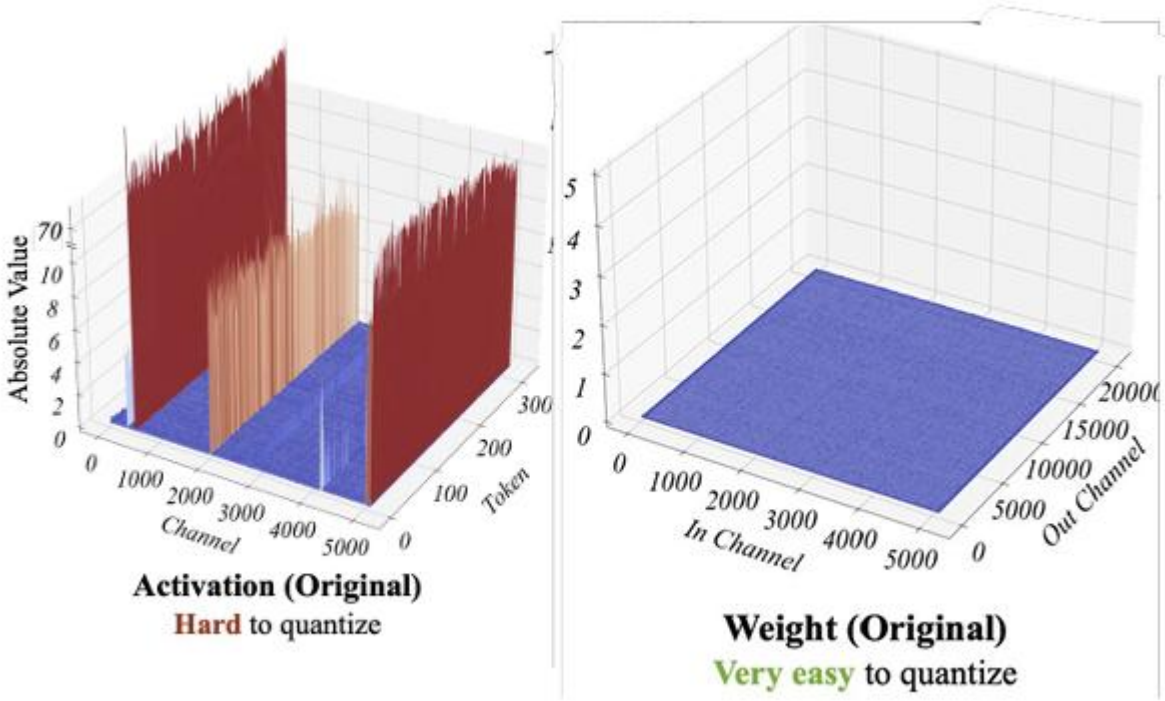
Activations are Intrinsically Challenging to Compress





- Output = Activation X Weight, so we want to quantize both weights and activations into INT8 (i.e., W8A8) to utilize the integer kernels (e.g., INT8 GEMM)
- Activation has many outliers (~70x larger) while the weight value is smooth (flat)
  - Outliers leave few effective bits for most values

SmoothQuant: Difficulty Transformation



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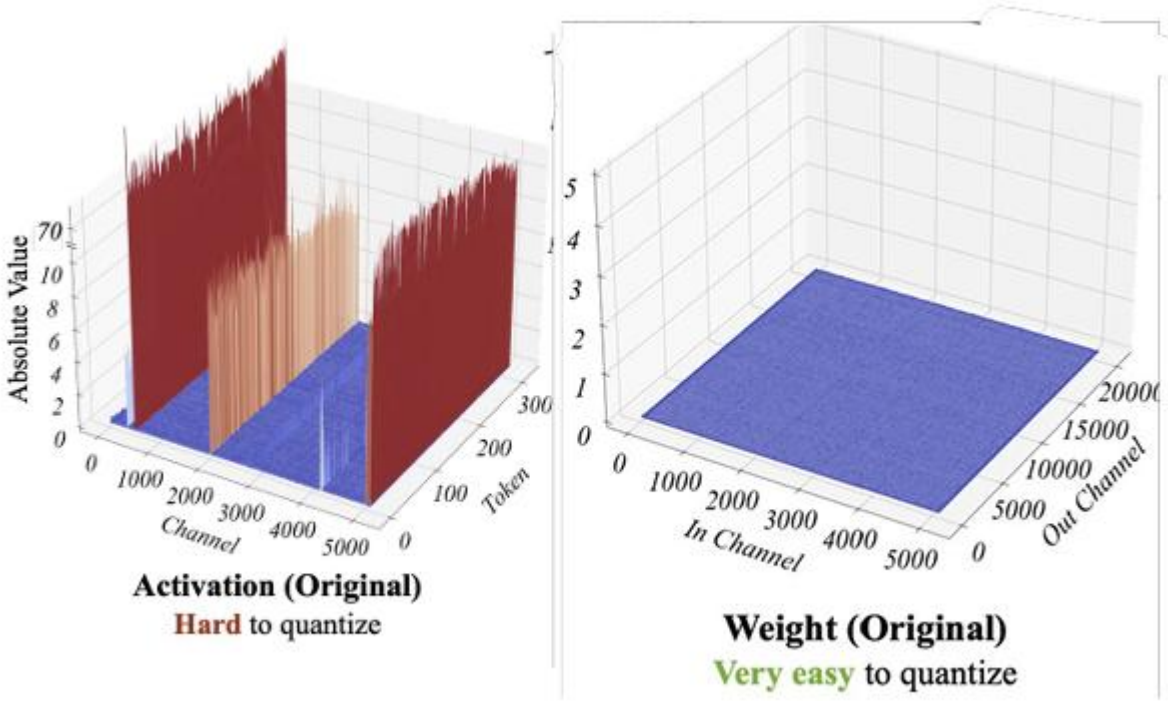
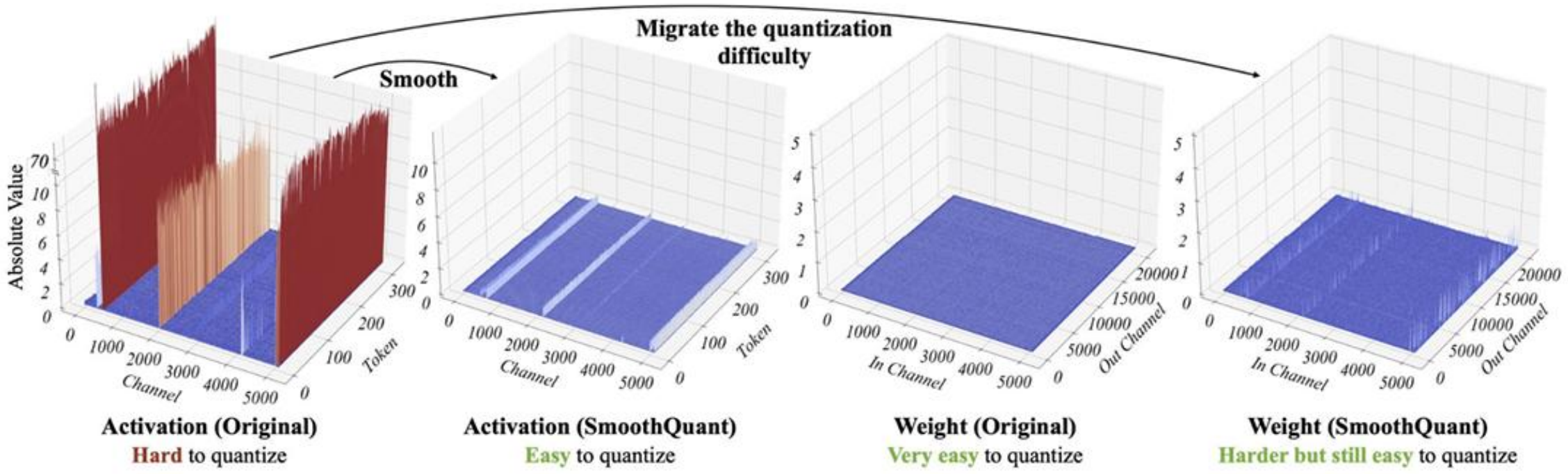


$$\mathbf{Y} = \mathbf{XW} = (\mathbf{0.01X})(\mathbf{100W})$$

- **Key idea: Migrating the quantization difficulty from activation to weight**

SmoothQuant: Difficulty Transformation



- **Key idea: Migrating the quantization difficulty from activation to weight**

SmoothQuant: Per-Channel Smoothing Factor

$$\mathbf{Y} = (\mathbf{X}\text{diag}(\mathbf{s})^{-1}) \cdot (\text{diag}(\mathbf{s})\mathbf{W}) = \hat{\mathbf{X}}\hat{\mathbf{W}}$$

- Push all quantization difficulty from activations to weights (all channels have same maximum magnitude):

$$\mathbf{s}_j = \max(|\mathbf{X}_j|), j = 1, 2, ..., C_i,$$

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- Push all quantization difficulty from weights to activations:

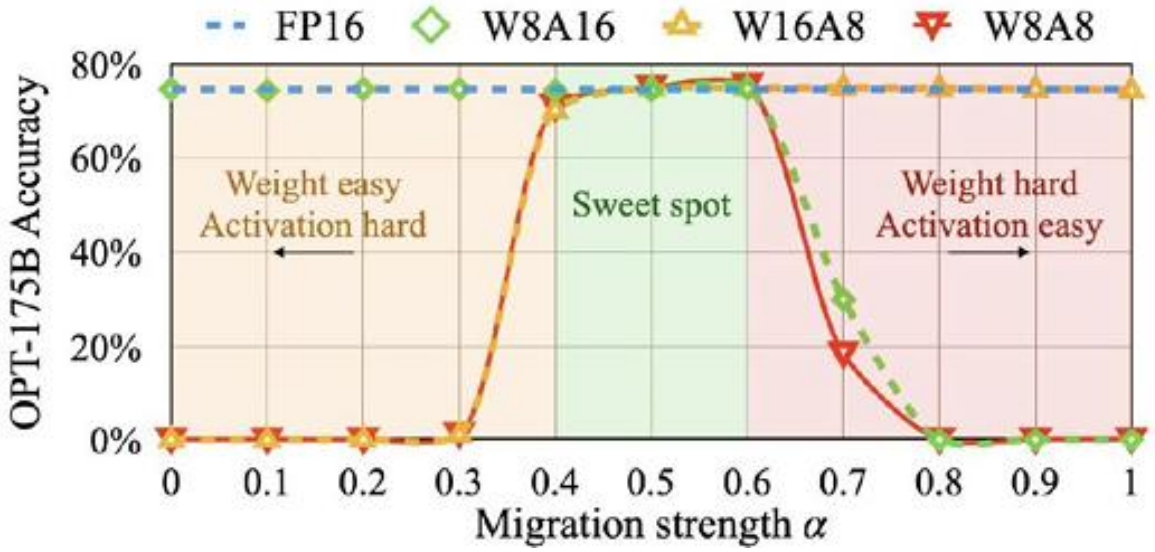
$$\mathbf{s}_j = 1/\max(|\mathbf{W}_j|)$$

- Balance difficulty according to α:

$$\mathbf{s}_j = \max(|\mathbf{X}_j|)^{\alpha} / \max(|\mathbf{W}_j|)^{1-\alpha}$$

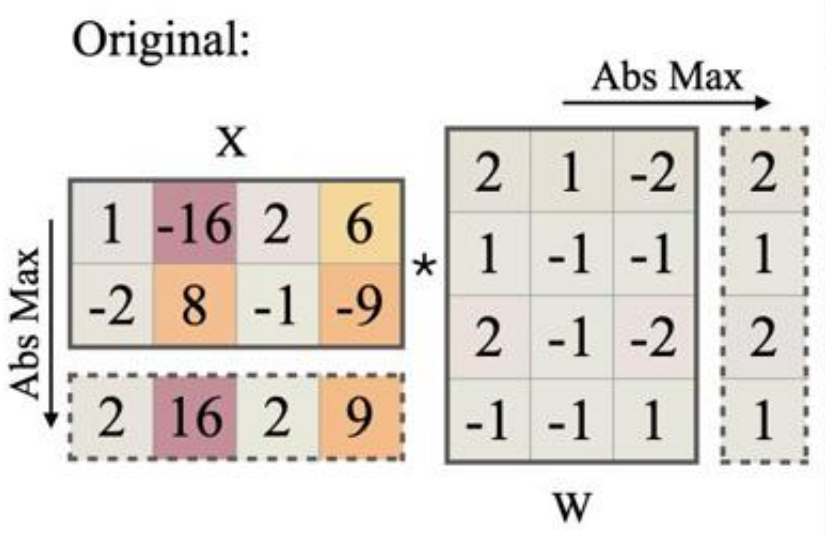
Choosing α

- Case-by-case decision
- If α is too large, weights will be hard to quantize. If too small, activations will be hard to quantize.
- Goal: Make activations and weights both easy to quantize.



Example of SmoothQuant

- 1) Applying Smoothing Factor
- 2) Quantize (constant step size)



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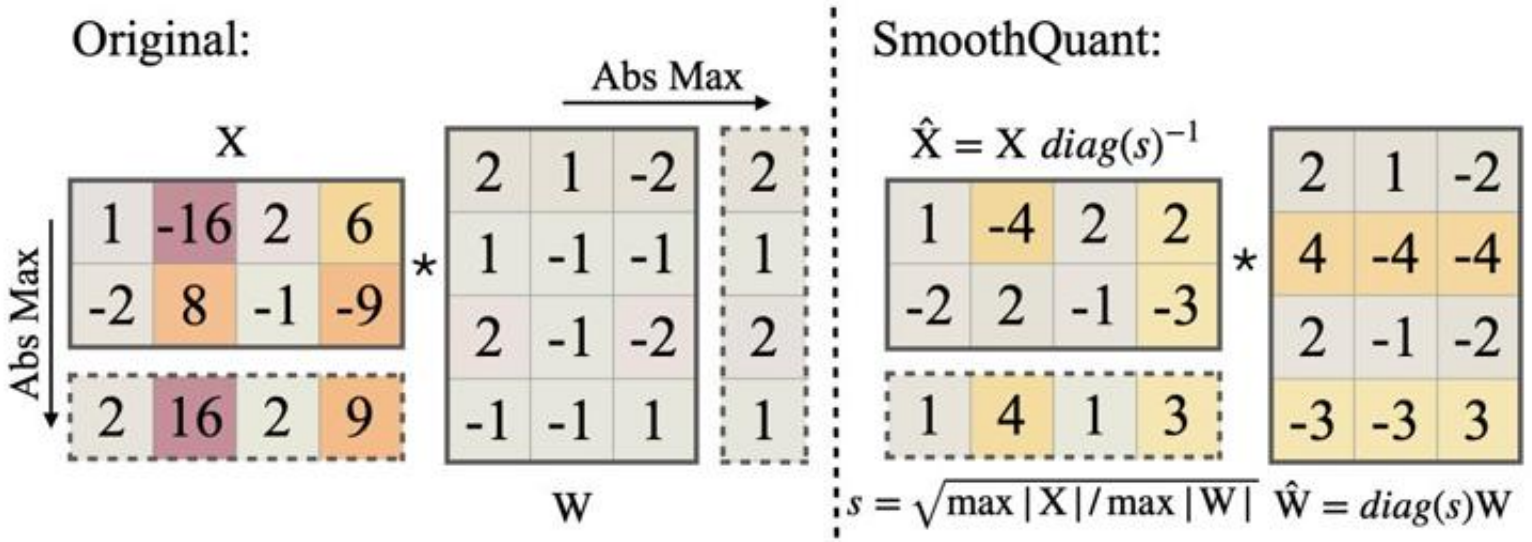
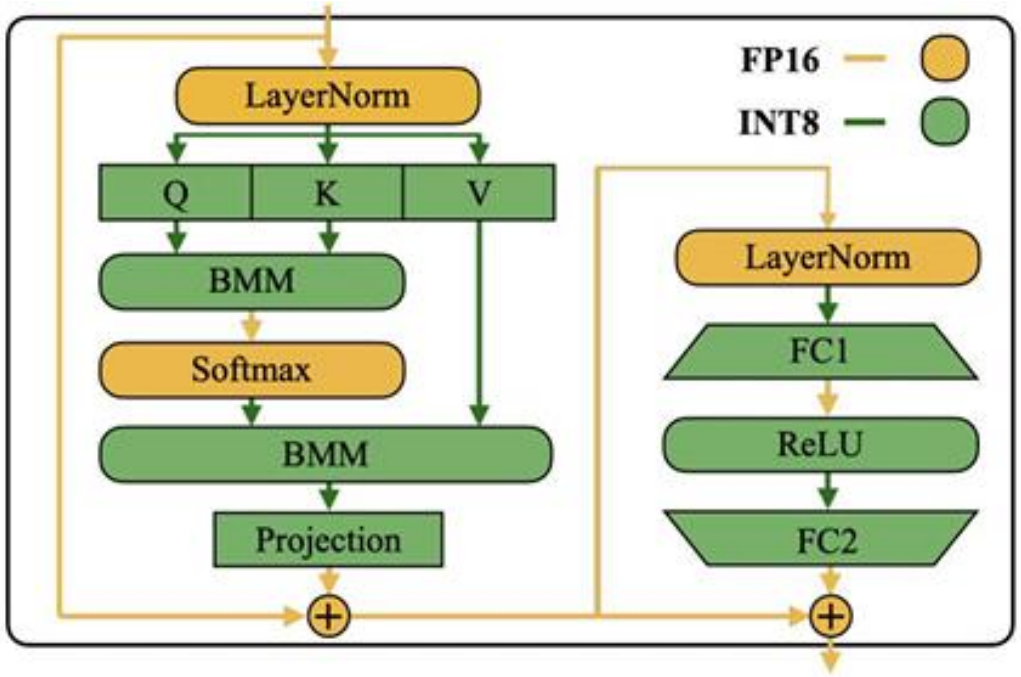
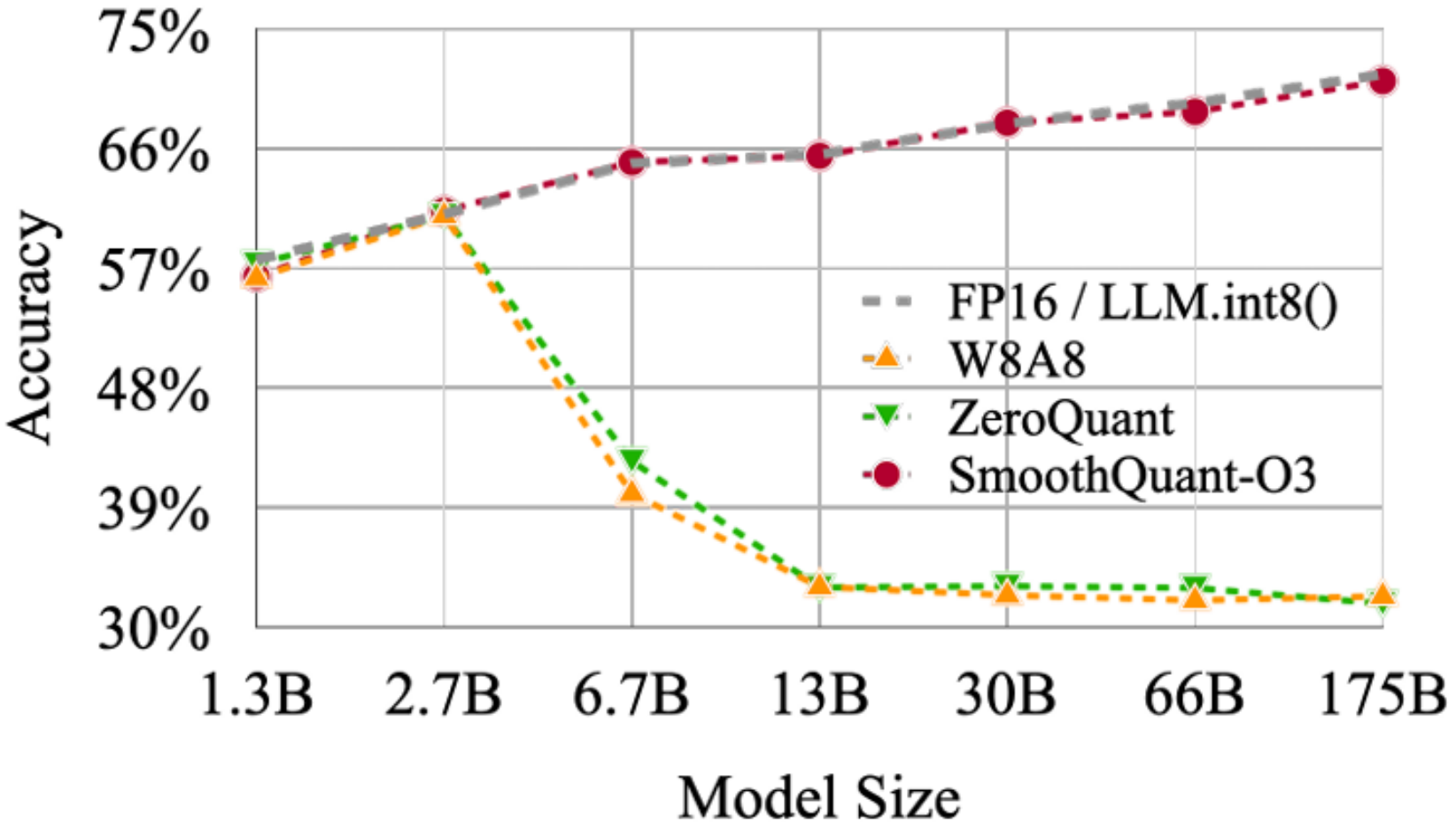


SmoothQuant Hardware Efficiency

- Applying SmoothQuant to transfer blocks
  - Linear layers take up most of the parameter and computation
  - Smoothing factor can be fused into previous layers' parameters offline
  - All linear layers are quantized with W8A8, as well as BMM operators in attention computation

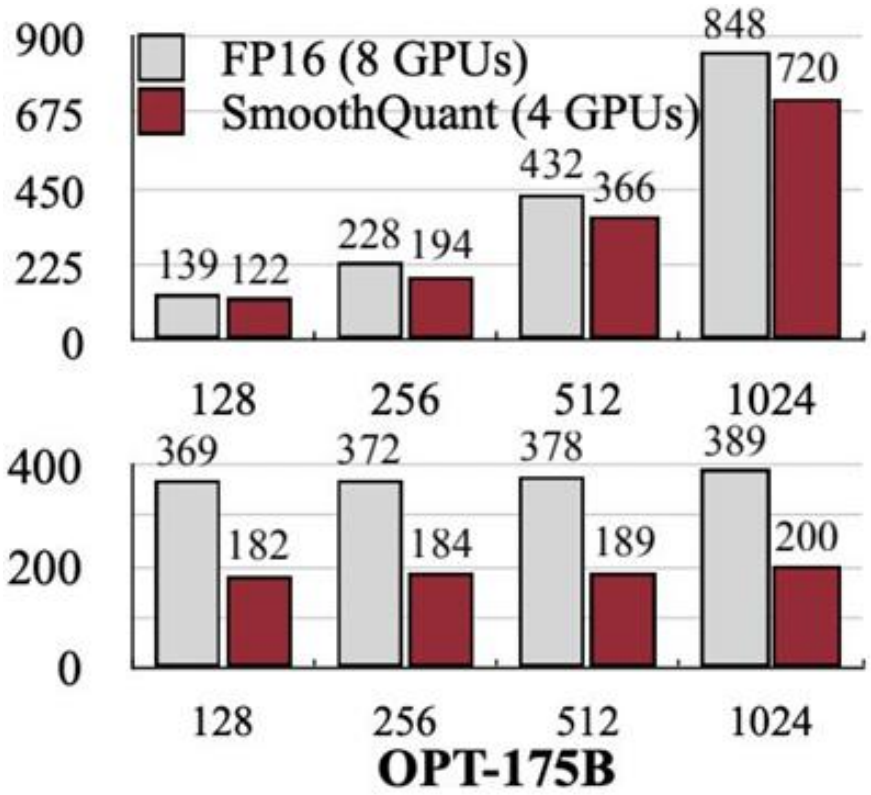


Evaluation: OPT 1.3B to 175B



Hardware Efficiency

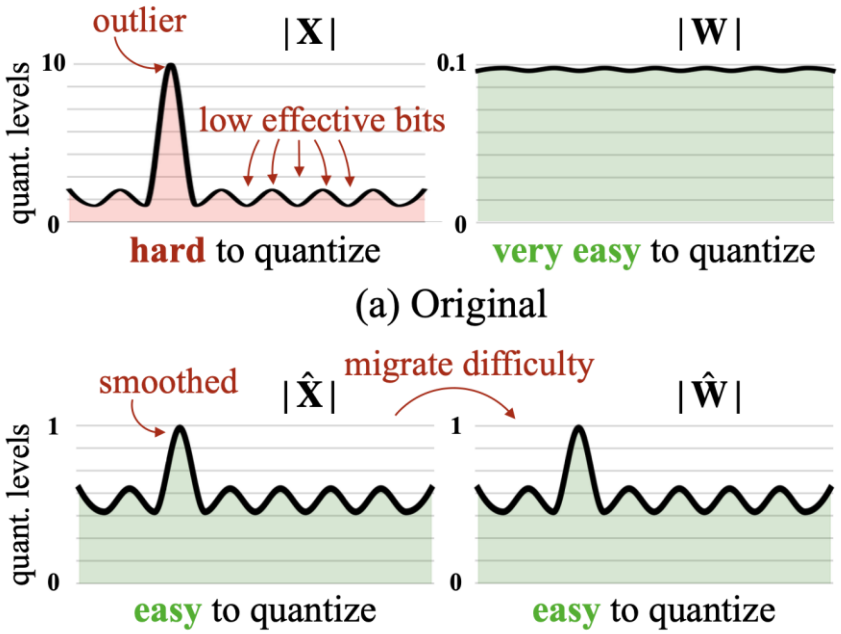
- Similar or faster latency with half #GPUs



Integrated into TensorRT and vLLM

Limitations

Limitations



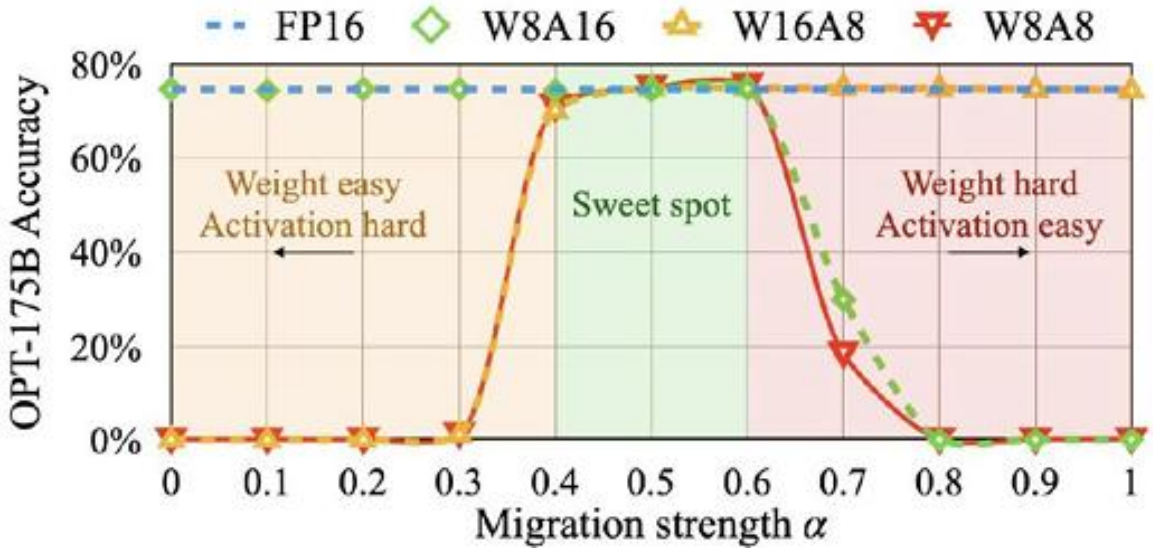
- Smoothing tradeoff
- Calibration dependency (e.g., smoothing factor)

Questions?

SmoothQuant: Alpha (α)

- Hyperparameter which controls the extent to which quantization difficulty is shifted from activations to weights

- α is between 0.4 to 0.6, though larger models or models with more significant activation outliers may require higher values.



Evaluation: SmoothQuant O1 to O3

- Gradually aggressive and efficient (lower latency) quantization levels

Method	Weight	Activation
SmoothQuant-O1	per-tensor	per-token dynamic
SmoothQuant-O2	per-tensor	per-tensor dynamic
SmoothQuant-O3	per-tensor	per-tensor static

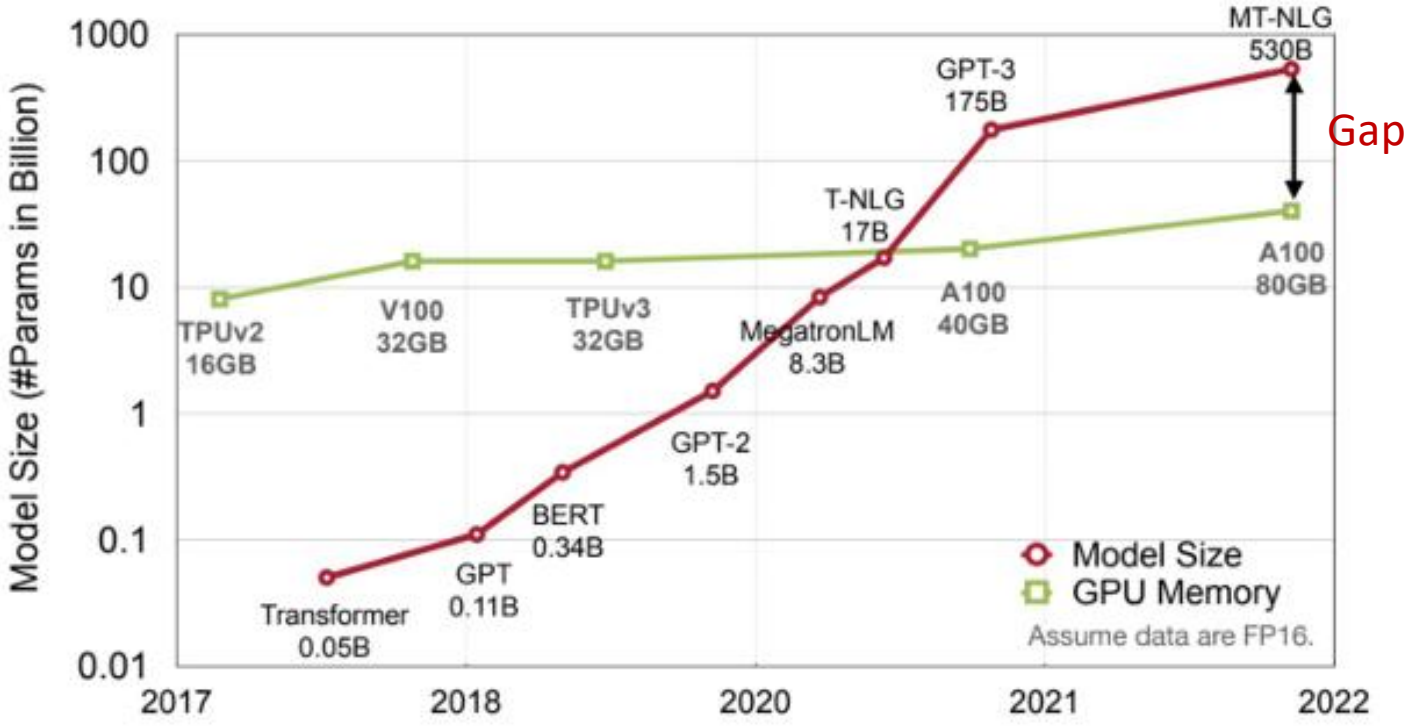
Serving Large Language Models Is Expensive

- Large language models (LLMs) are taking over every field.

- As the models get larger, serving such models for inference becomes **expensive** and **challenging**!

Gap Between the Supply and Demand of LLMs

- Size of LLMs is increasing faster than GPU memory: **exponential growth in model size** vs. linear increase in GPU memory



Gap Between the Supply and Demand of LLMs

- Size of LLMs is increasing faster than GPU memory: **exponential growth in model size** vs. linear increase in GPU memory
- **Memory challenges:** 175B parameters in GPT-3 requires 350GB of memory to store just weights